Optimal Repeat Inspection Plan with Several Classifications

by

Mehmood Khan

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

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DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

SYSTEMS ENGINEERING

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To my dear parents

Acknowledgments

All praise be to Allah, the Lord of all the worlds. May peace and blessings be upon Mohammad the last of the messengers. I am thankful to Allah for His limitless help and guidance.

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التخصص الدقيق	هندسة نظم
التأريخ	مايو/أيار ٢٠٠٠

ادعى تزايد الوعي بدور المبيعات وتكلفة الأعطال على التركيز على جودة المنتج كهدف استراتيجي رئيسي للشركات الصناعية. وينطبق ذلك بصورة أكبر على الشركات التي تنتج منتجات عالية التقنية. ويدرس هذا البحث فحص القطع الحرجة وهي القطع التي يسبب تعطلها كارثة أو تكلفة كبيرة. ويكون للقطعة عدة أجزاء وتعطل أي جزء منها أو مخالفته للمواصفات يؤدي إلى رفض القطعة. ولذا لا بد من فحص هذه القطع عدة مرات أو ما يسمى بالفحص المكرر وأدى هذا إلى بروز خطط الفحص المكرر كوسيلة مهمة لضبط الجودة.

في هذه الأطروحة اقترحت خطة عامة للفحص المكرر وطور نموذجين لتمثيل هذه الخطة. أحدهما عندما تكون احتمالات الخلل في أجزاء القطعة غير معتمدة على بعضها والآخر عندما تكون هذه الاحتمالات معتمدة على بعضها. تطور هذه النماذج البحوث السابقة في هذا المجال لأنها تتضمن عدة أنواع من الأخطاء التي يمكن أن يرتكبها الفاحص. كما أجريت دراسة عن تأثير أخطاء الفحص على مقاييس أداء الخطة في حالة النموذجين. وتم مقارنة النموذجين على أساس هذه الدراسة.

ماجستير العلوم

جامعة الملك فهد للبترول والمعادن

الظهران، المملكة العربية السعودية

مايو/أيار ٢٠٠٠

THESIS ABSTRACT

FULL NAME	MEHMOOD KHAN
TITLE OF STUDY	OPTIMAL REPEAT INSPECTION PLAN WITH SEVERAL CLASSIFICATIONS
MAJOR FIELD	SYSTEMS ENGINEERING
DATE OF DEGREE	MAY 2000

Increasing awareness of the sales and cost of repair implications has led to the emergence of product quality as a major strategic instrument for industrial corporations. This is especially true for firms producing high technology products. The components, on the other hand, may have many characteristics, nonconformance of one of which can result in the failure of a component. In case the components are critical, this failure can be catastrophic. While inspecting for a defective, an inspector can make several false classifications. Consequently, repeat inspection plans have become increasingly important in the area of quality control to minimize the inspection error.

In this thesis, a general inspection plan is proposed. A cost minimization model that depicts the plan is developed. The model is more general than the ones existing in the literature because it includes several types of classification errors. Then, the model is extended for the case where characteristics' defective rates are statistically dependent. A detailed study of the effect of inspection error on the repeat inspection plan for the two models is conducted. Finally, the results of the two models are compared on the basis of this study.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Quality Control is becoming the basic consumer decision factor in many products and services. This phenomenon is widespread, regardless of whether the consumer is an individual, an industrial corporation, a military defense program or a retail store. Consequently, quality is a key factor leading to business success, growth, and enhanced competitive position. There is a substantial return on investment from an effective quality-improvement program that provides increased profitability to firms that effectively employ quality as a business strategy. Effective quality-improvement programs can result in increased market penetration, higher productivity, and lower overall costs of manufacturing and service.

Statistical quality control is basically comprised of two areas: Process control and Product Control. The product control can be accomplished by

a) Acceptance sampling plans

b) Complete inspection plans

Acceptance sampling is likely to be used under the conditions where:

(i) The cost of inspection is high and the loss arising from passing a defective unit is not great.

(ii) The 100% inspection is fatiguing and a carefully worked-out sampling plan will produce as good or better results.

(iii) The inspection is destructive. In this case, sampling must be employed.

Complete inspection plans are becoming increasingly important in the area of quality control due to the developments in modern manufacturing systems. The modern means of science and technology have proved that complete inspection, though complex, is possible and quite applicable. Complete inspection plans are useful in the situations where the loss arising from passing defective component is high or catastrophic. Next section explains the inspection tasks, types of errors and their applications. Section 3 gives the objectives of the thesis and the following two sections briefly describe how these objectives would be achieved and organized in the thesis.

1.2 Inspection Tasks

Inspection can be defined as the function of comparing or determining the conformance of products to established specifications. Inspection tasks may be classified into three basic categories, these are: tasks involving visual scanning, tasks involving measurements, and tasks involving monitoring of a process.

The area of concern in this thesis is that of industrial inspection where the inspector error can cause extensive damage by failure to identify defective material. If such material is used in subsequent assembly operations, it may cause serious damage to material or injury to personnel. Although the technology of industrial inspection has made rapid advances in the past few years, there is still too much waste resulting from inspector error. In addition ignoring the presence of inspection error can severely distort the performance measures of any inspection activity. It is not unusual to find inspection error rates of 25% or 30% in complex inspection activities [19].

1.2.1 Accuracy of Inspection

The inspection accuracy is influenced by number of factors [19]. These factors can be categorized into three major groups:

1. Inspector related factors, such as the age, experience, sex, visual activity, intelligence, level of training, psychological factors etc.
2. Task related factors, such as task pacing, task perception, task complexity, design of work place, rate of defects etc
3. Environmental and organizational factors, such as, illumination, noise temperature, humidity, motivation, incentives etc.

1.2.2 Type I and Type II Errors

Inspectors make two types of errors. One is the type I error, in which a conforming quality characteristic is classified as nonconforming; the other is the type II error, in which a nonconforming characteristic is passed as conforming. This is presented in Table 1.1.

Table 1.1: Decision Matrix

	Decision Based on Inspection	
	Accept	Reject
Conforming	Correct Decision	Wrong decision Type I error
Nonconforming	Wrong decision Type II error	Correct Decision

1.2.3 Critical Components

A critical component is taken to be the one, which, upon failure, may cause serious damages like loss of human life or a very high cost. These components may be a part of:

1. A nuclear reactor
2. An aircraft
3. A space shuttle

Critical components normally have many characteristics, defect in any of which may cause serious loss. To guard against these losses, we need to inspect each of these characteristics repeatedly. Repeat Inspection is useful in overcoming the effect of type I and type II errors committed by the inspectors.

The models in the literature are based on two inspection plans given by Raouf et al [25] and Duffuaa and Al- Najjar [14]. They are all developed on the assumption that there is just one class of type I and type II error each. While in the usual case of inspection, an inspector does not only classify a component to be defective or non-defective with respect to certain characteristic. But in many situations other classifications may arise. For example an inspector may classify a characteristic to be acceptable or to be reworked or scrapped. This gives rise to a number of classification errors. For example, a good characteristic could be misclassifed as rework or defective or a defective characteristic could be classified as good or rework. A need exists to develop a general repeat inspection plan and a model to represent these several classifications.

The main objective of this thesis is to extend the model in Raouf et al [25] to incorporate several classes of errors an inspector can make. The model will then be modified to the case where the characteristic defective rates are dependent.

1.3 Objective of the Thesis

The overall objective of this thesis is to generalize and extend the inspection plan and the models in the area of repeat inspection. Specifically, the objectives of this thesis are:

1. To extend the multicharacteristic repeat inspection model to incorporate several classification errors.
2. To investigate the effect of inspection errors in the extended model.
3. Extend the model developed in (1) to the case of dependent characteristics.
4. To investigate the effect of inspection errors for the model in (3).
5. To compare the results of the model in (1) with the one in (3).

1.4 Plan of the Proposed Work

The research work proposed for the thesis extends the model developed by Raouf et al [25]. It will begin with extending the above model for the case where an inspector can commit several types of misclassifications. A computational procedure will be outlined to determine the optimal number of inspection cycles in the new model. The effect of sequencing on the inspection plan is studied by comparing the results of the model for two cases viz (i) an optimal sequence and (ii) a fixed sequence of characteristics for each cycle of inspection.

Then the new model will be extended for situations where the characteristics' defective rates are statistically dependent. This requires the knowledge of the joint probability mass function (j.p.m.f.) of the random variables representing characteristics' defective rates. Using (j.p.m.f.) we can obtain the individual marginal probability mass function. Since the joint and the marginal mass function varies from cycle to cycle, the values for the individual random variable marginal mass function will be updated using Bayes' theorem. After the inspection of first characteristic, the marginal of other characteristics must be updated prior to inspecting them. Using these concepts, the model will be extended to handle the case where characteristics defective rates are statistically dependent. At last, a comparison of the results obtained with and without consideration of the statistical dependency of the defective rates will be presented.

1.5 Thesis Organization

The thesis is presented in six chapters. Following the introduction in the first chapter, chapter 2 presents some background on the inspection tasks and then an extensive review of the literature. There are three main classes of articles in the literature review. The first class is those articles that deal with the role of the human psychology on the performance inspection tasks. The second class of articles deals with sampling plans and the third are those which deal with repeat inspection plans for the multicharacteristic critical components.

Chapter 3 proposes a new repeat inspection plan in the thesis. The details of the model based on the plan are described. Computational procedure is supplied together with a numerical example to estimate the optimal number of the inspection cycles.

In chapter 4, the model in chapter 3 is modified to allow the dependency between the characteristics' defective rates. In this a chapter a computational procedure is supplied together with a numerical example to estimate the optimal number of the inspection cycles. A comparison between the model in chapter 3 and the one in this chapter are made.

In chapter 5, the effect of inspection error on the performance measures of the inspection plan is made. This is carried out by observing the behavior of ETC, AOQ and ATI at different levels of type I and type II errors. Chapter 6 provides the conclusions and future research directions.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Inspection is the component of the quality control program that is concerned with checking on the conformance of the item to the specifications set for it. The inspection commonly fulfills four primary responsibilities: (1) it checks the quality of incoming materials, (2) it checks on all finished goods to insure that only acceptable products reach the customer, (3) it aids in maintaining process control and attempts to locate the flaws in manufacturing that would cause subsequent difficulties, and (4) it serves in an advisory capacity in attempting to correct or prevent quality control problems.

Embedded within the design of many inspection tasks is an assumption that the inspection procedures are error free. However, many inspection tasks are not error free; on the contrary, they may even be error prone. The errors that result from inspection operations have the effect of severely distorting the performance measures of any inspection activity whose design has ignored their presence. It is not unusual to find inspection error rates of 25% or 30% in complex inspection activities. Components whose failure result in catastrophic or serious hazard or which result in a very high cost are termed critical components. Such components have many characteristics that must conform to tight product specifications. Examples of such components could be a part of a gas ignition system, a space shuttle, an aircraft or a nuclear reactor.

To guard against a serious hazard, repeat inspection is usually instituted to reduce inspection errors i.e. type I and type II errors. The first inspection plan for critical components was developed in [25]. It involved the sequential inspection of all characteristics in a component. The process was repeated until the optimal number of inspections. Later, the plan in [25] has been modified in [11] and several models for this plan have been developed and found to be superior to the model in [25].

The purpose of this chapter is to present a concise review of the literature relevant to this thesis.

2.2 Literature Review

Ayoub et al [3] defined mean inspection error to be the average number of defectives classified as good items by the inspector. They gave a formula for Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) from a single sampling plan under inspection error. Adams [1] summarized some of the perceptual, statistical modeling and behavioral aspects in industrial inspection. It was suggested that Signal Detection Theory (SDT), behavioral modification technology and improved definitions of observing responses be involved in this approach. Collins et al [7] considered the effects of inspection error on probability of acceptance, average outgoing quality and average total inspection. They examined these measures under both replacement and non-replacement assumptions. Bennet et al [4] investigated the effect of error on a single sampling plan with known incoming quality. Shor and Raz [26] reported a study to identify and rank the human factors that have the greatest input on the occurrence of inspection errors. Sylla and Drury [28] proposed a model, which uses a form of SDT to predict inspector performance in order to improve system performance. They gave the concept of lability to characterize the inspector's ability to respond to costs, penalties and probabilities involved in the inspection decision.

Mei et al [22] gave the design of acceptance sampling plans for a general lot distribution with known and constant variance. They showed that bias and imprecision inspection have significant effect on Operating Characteristic (OC) Curve. A method presented whereby the variable sampling plan may be designed to explicitly compensate for measurement error and provide the desired OC curve. Dhavale [8] suggested that the distribution of defectives in 100% inspected lots follows negative binomial distribution. He assumed that Poisson distribution describes the inspector errors while Gamma distribution reflects the diversity among the inspectors. Maghsoodloo [21] provided the expressions for the performance measures, probability of acceptance P_a, AOQ and ATI in presence of inspector error in a multistage sampling plan. Suich [27] considered the effects of inspection error on acceptance sampling while inspecting for the number of non-conformities per item He investigated the effect of inspection error on both OC curves and on rectifying inspection procedures.

Raouf et al [25] were the first to develop a model for determining the optimal number of repeat inspections for multicharacteristic components to minimize the total expected cost per accepted component due to Type I error, Type II error and cost of inspection. Tang and Schneider [29] investigated the economic and statistical effects of inspection error on the complete inspection plan. They developed two models with considerations of inspection error under different rework schemes and then compared to the model without inspection error consideration. Jaraiedi et al [18] gave a model to determine the AOQ for a product which has multiple quality characteristics and which is subject to multiple 100% inspections where the inspection is subject to errors. Tang and Schneider [30] developed a model for determining the most economical inspection precision level for 100% inspection. They assumed the inspection cost to be a linear function of the inspection precision level. Lee [20] provided a simplified version of the model given by Raouf et al [25] to evaluate the costs in the multiple-cycle inspection schemes for multicharacteristic components. He extended the results for the case where the probabilities of defectives are random. Chandra and Schall [5] studied the effect of replicate measurements on average outgoing quality and the average total inspection. They obtained the optimum number of replications based on total cost of inspection. Tang and Tang [31] extended the economic model for a screening procedure using more than one correlated variable. They improved the screening accuracy by increasing the correlation between the linear combination of the correlated variables and the performance variable. Duffuaa and Raouf [11] developed three mathematical optimization models for multicharacteristic repeat inspection. The first model (cost minimization model) minimizes the total cost due to inspections, Type I error and Type II error to determine the optimal number of repeat inspections. The second model (probability minimization model) minimizes the probability of accepting a defective component. The third model (satisfying model) determines a satisfying solution by specifying an upper limit for total inspection cost and for the probability of accepting a defective component. Duffuaa and Raouf [12] established an optimal rule for sequencing characteristics for inspection in the plan proposed by Raouf et al [25]. Yumei and Tang [33] proposed two models with different screening procedures to determine the acceptability of produced items. They used Taguchi's quadratic loss function to determine the loss due to deviation from the product target. Hui [17] studied the complete inspection plan for bicharacteristic products in variable inspection. A decision approach is adopted to determine the best strategy after inspection. Duffuaa and Nadeem [13] proposed a new inspection plan for critical multicharacteristic components. They proposed an algorithm to determine the optimal number of repeat inspections and sequenced the characteristics for inspection in order to minimize the total expected cost. Tang and Tang [32] reviewed the literature in the area of screening. They suggested that screening should be considered only as a short-term method to remove non-conforming items from a population of items. Duffuaa and Al-Najjar [14] proposed a new inspection plan for critical multicharacteristic components. They proposed an algorithm to determine the optimal number of repeat inspections and sequence characteristics for inspection in order to minimize the total expected cost. Duffuaa [10] investigated the statistical and economic impact of the inspector errors on the performance measures, i.e. ATI, AOQ and ETC of a complete inspection plan. He concluded that type I and type II errors have significant effect on the performance measures of repeat inspection plans. Drezner and Wesolowsky [9] extended the work of Tang and Tang [31] dealing with quality cost of a process that is screened with the use of correlated variables. They give general set of loss functions to model both traditional and contemporary losses for deviation from target values or product specification. Chen and Labrecht [6] proposed a model to optimize the sequence and frequency of inspections of multicharacteristic components. They used marginal analysis and gave an efficient algorithm to find the optimal plan. Hong and El Syed [15] developed economic complete inspection plans for determining the best market in situation where there are several markets with different price/cost structures. They considered the complete inspection based on performance variable of interest and the screening variable, which is correlated with performance variable. Hong et al [16] developed economic screening procedures when the rejected items are reworked. Screening procedures based on the performance variable of interest and a correlated variable are considered. They considered the cost incurred by imperfect quality, reprocessing cost and inspection cost.

From the review of the literature, it can be inferred that all the models in repeat multicharacteristic inspection were developed on the basis of the plans given by Raouf et al [25] and Duffuaa and Najjar [14]. The plans in the literature take just one form of the type I and type II errors. A need exists to a new inspection plan, which allows a very general characterization of the product to be made by the inspector, i.e. non-defective, to be reworked or to be scrapped, with respect to a certain characteristic.

This thesis proposes this new plan for multicharacteristic inspection and develops two optimization models, which depict the proposed plan.

CHAPTER 3

MATHEMATICAL MODEL FOR THE INDEPENDENT CASE

3.1 Introduction

In this chapter, a new inspection plan for the multicharacteristic repeat inspection of the critical components is presented. The development of the model is based on the fact that there can be several types of misclassification errors an inspector can make.

The plan in the literature is as follows. Each inspector inspects one characteristic and classifies the component to be defective or nondefective. The accepted components are passed onto the next inspector. This process is repeated until all the characteristics are inspected. All the accepted components are sent back to the first inspector and the process is repeated until the required number of inspection cycles is complete.

The inspection plan and the model representing the plan are developed for components having several characteristics with known incoming quality. This model is an extension of the model given by Raouf et al [25]. A component is classified non-defective only if all the characteristics meet the quality specifications. The inspection plans for this thesis assume three classes for the components under inspection, viz. to be accepted, to be reworked and to be scrapped. The probabilities of misclassifications by the inspector (i.e.

Type I and Type II error) are assumed to be known. Three types of costs are considered: (i) cost of false rejection of an acceptable component (sent to rework or to scrap), (ii) cost of false acceptance of the components which are to be reworked or to be scrapped, and finally, (iii) cost of inspection. The cost of inspection is taken to be of two types i.e. for the inspection at the inspection station and for the inspection at the rework station. The estimates for the three costs are assumed to be available in the industry.

The inspection plan is shown in Figure 3.1 and is applied as follows: an inspector inspects one particular characteristic for each component entering the inspection process and classifies them as the accepted ones, scrapped or as rework required. An expensive and error free inspection is assumed to be made at the rework station, which separates the good and scrap items falsely going to rework. All the components accepted by the inspector and the ones that are found to be good at the rework station go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and the process is repeated a total of n times. Here n is the optimal number of inspection necessary to minimize the cost per accepted component. Finally, the accepted components will be those which are accepted in n-th cycle, and the total scrapped components will be the sum of those scrapped in the 1st, 2nd,..., n-th cycles. In the next two sections the inspection model is described and developed. A computational procedure for the inspection plan with an example is given sections 4 and 5 respectively. Section 5 describes the effect of sequencing on the inspection model. Section 6 concludes the chapter.

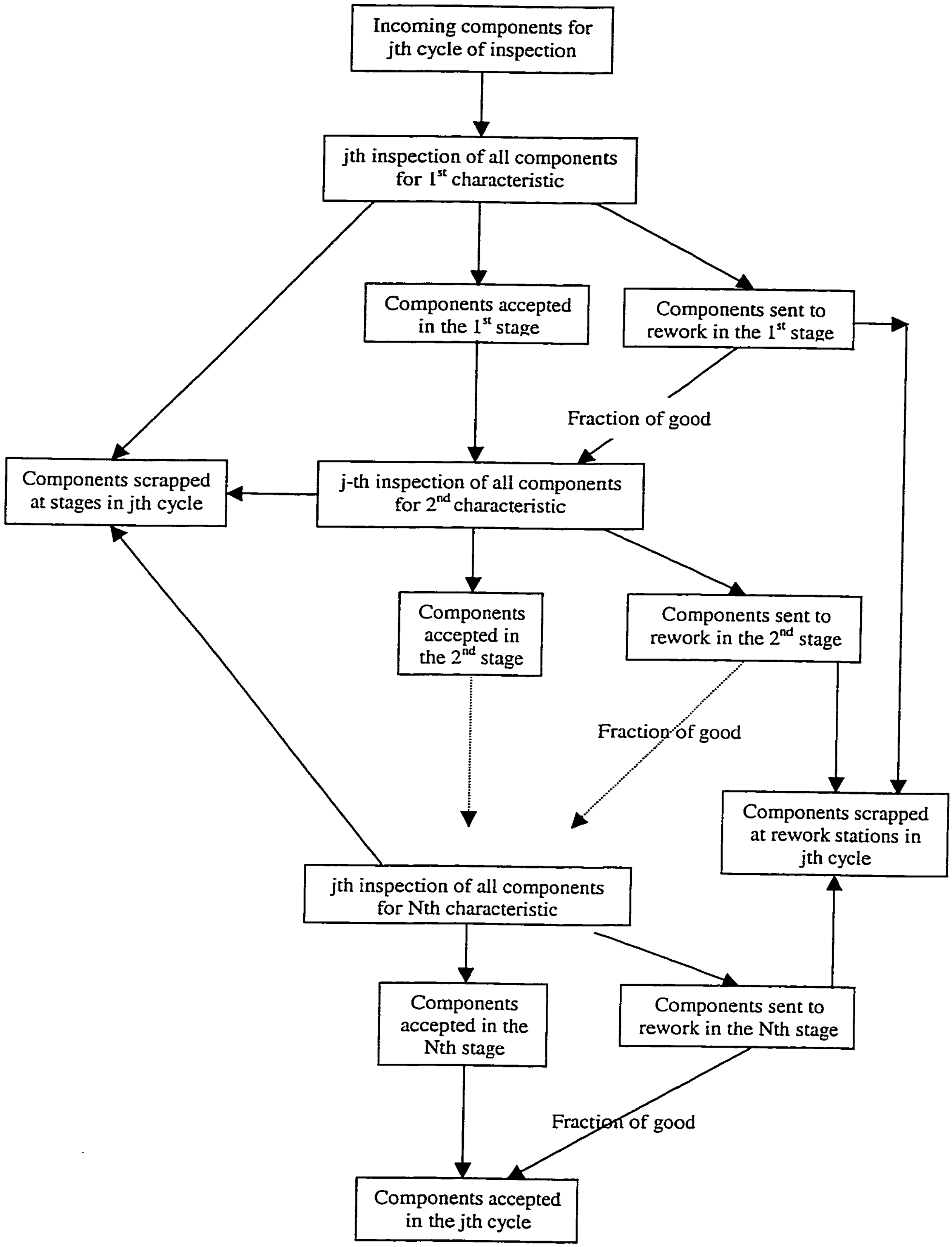

Fig. 3.1 Inspection Plan for jth Cycle.

3.2 Model Description

The model is developed for components having several characteristics. The incoming quality of the components i.e. the probabilities of being good, scrap or to be reworked is assumed to be known. A component is classified as non-defective only if all the characteristics meet the quality specifications. The probabilities with which the inspector commits misclassifications are assumed to be known. Three different types of costs are considered: (i) cost of false rejection of good items (i.e. sending them to either to scrap or for rework), (ii) cost due to false acceptance of an item which is either reworkable or to be scrapped, and, finally, (iii) cost of inspection both by the inspector and at the rework station. The equipment for inspection at the rework station is taken to be error free and is therefore very costly. The estimates for these costs are assumed to be available.

The inspection plan is as follows: an inspector inspects one particular characteristic for each component entering the inspection process, and classifies them to be acceptable, scrap or to be reworked. All the accepted components and the ones that are found to be good at rework station, go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and this process is repeated a total of n times before the components are finally accepted. Here n is the optimal number of inspections necessary to minimize the total cost per accepted component. Finally, the accepted components will be those that are accepted in the nth cycle, and the total scrapped components will be the sum of those scrapped in the 1^{st}, 2^{nd}, …., nth cycles. Next the notations used are given.

Notations:

M_j = Number of components entering the jth cycle of inspection

N = Number of characteristics in each component to be inspected.

${}^{j}P_{is}$ = probability of ith characteristic in the sequence of inspection being scrap while entering the jth cycle of inspection .

${}^{j}P_{ir}$ = probability of ith characteristic in the sequence of inspection being rework while entering the jth cycle of inspection.

${}^{j}P_{ig}$ = probability of ith characteristic in the sequence of inspection being good while entering jth cycle of inspection.

${}^{j}PS$ = Probability of a component being scrap while entering the jth cycle of inspection.

${}^{j}PR$ = Probability of a component being rework while entering the jth cycle of inspection.

${}^{j}PG$ = Probability of a component being good while entering the jth cycle of inspection.

E_{igr} = Probability of classifying the ith good characteristic in the sequence of inspection as rework.

E_{igs} = Probability of classifying the ith good characteristic in the sequence of inspection as scrap.

E_{irg} = Probability of classifying the ith reworkable characteristic in the sequence of inspection as good.

E_{irs} = Probability of classifying the ith reworkable characteristic in the sequence of inspection as scrap.

E_{isg} = Probability of classifying the ith scrappable characteristic in the sequence of inspection as good.

E_{isr} = Probability of classifying the ith scrappable characteristic in the sequence of inspection as rework.

$M_{i,j}$ = Number of components entering the ith stage of inspection in jth cycle.

$PG_{i,j}$ = Probability of a component being good in the ith stage of the jth cycle .

F_{iab} = Number of components of type a misclassified as type b in ith stage.

$FGR_{i,j}$ = Number of good components falsely sent to rework in ith stage of the jth cycle

$FR_{i,j}$ = Number of components falsely sent to rework in ith stage of the jth cycle .

$FS_{i,j}$ = Number of components falsely sent to scrap in ith stage of the jth cycle. .

$FA_{i,j}$ = Number of falsely accepted components in the ith stage of the jth cycle.

$CA_{i,j}$ = Number of correctly accepted components in the ith stage of the jth cycle.

$R_{i,j}$ = Rate of rejection of components due to ith characteristic in the sequence of inspection in jth cycle.

A(j) = Number of accepted components in jth cycle.

CFR(j) = Cost of false rejection in jth cycle.

CFA(j) = Cost of false acceptance in jth cycle.

CI (j) = Cost of inspection in jth cycle.

TCFR = Total cost of false rejection.

TCFA = Total cost of false acceptance.

TCI = Total cost of inspection.

TA = Total number of accepted components.

E(tc)|j = Expected total cost per accepted component after j cycles of inspection.

E( ) = Expected value of the argument inside the parentheses

3.3 Model Development

The probability of ith characteristic being good, scrap or rework varies from cycle to cycle. These three probabilities for the incoming components are assumed to be known. First, we shall establish the relationship between the incoming quality and the quality in the next cycles of inspection.

Expressions for $^{j}P_{ig}$, $^{j}P_{ir}$ and $^{j}P_{is}$

We can write the incoming probabilities for the different classes as

$$^{1}P_{ig} = P_{ig}, {}^{1}P_{ir} = P_{ir}, {}^{1}P_{is} = P_{is} \tag{3.1}$$

In the sequence of inspections, the probability of an item going into the second stage is

$$\mathrm{P}_{ig}(1 - E_{igr} - E_{igs}) + \mathrm{P}_{ig}E_{igr} + \mathrm{P}_{ir}E_{irg} + \mathrm{P}_{is}E_{isg}$$

or

$$\mathrm{P}_{ig}(1 - E_{igs}) + \mathrm{P}_{ir}E_{irg} + \mathrm{P}_{is}E_{isg}$$

So, the probabilities of the components having different classifications entering into the second stage are

$$^{2}\mathrm{P}_{ig} = \frac{\mathrm{P}_{ig}(1 - \mathrm{E}_{igs})}{\mathrm{P}_{ig}(1 - \mathrm{E}_{igs}) + \mathrm{P}_{ir}\mathrm{E}_{irg} + \mathrm{P}_{is}\mathrm{E}_{isg}} \tag{3.2}$$

$$^{2}P_{ir} = \frac{P_{ir}E_{irg}}{\mathrm{P}_{ig}(1 - E_{igs}) + \mathrm{P}_{ir}E_{irg} + \mathrm{P}_{is}E_{isg}} \tag{3.3}$$

$$^{2}P_{is} = \frac{P_{ig}E_{isg}}{\mathrm{P}_{ig}(1 - E_{igs}) + \mathrm{P}_{ir}E_{irg} + \mathrm{P}_{is}E_{isg}} \tag{3.4}$$

So, for the third stage, we can write

$$^3P_{ig} = \frac{^2P_{ig}(1-E_{igs})}{^2\mathrm{P}_{ig}(1-E_{igs})+{}^2\mathrm{P}_{ir}E_{irg}+{}^2\mathrm{P}_{is}E_{isg}}$$

Substituting the values of $^2P_{ig}, {}^2P_{ir}, {}^2P_{is}$ from (2), (3) and (4) above gives, after simplification,

$$^3P_{ig} = \frac{P_{ig}(1-E_{igs})^2}{\mathrm{P}_{ig}(1-E_{igs})^2 + \mathrm{P}_{ir}E^2{}_{irg} + \mathrm{P}_{is}E^2{}_{isg}} \tag{3.5}$$

Similarly,

$$^3P_{ir} = \frac{P_{ir}E_{irg}{}^2}{\mathrm{P}_{ig}(1-E_{igs})^2 + \mathrm{P}_{ir}E^2{}_{irg} + \mathrm{P}_{is}E^2{}_{isg}} \tag{3.6}$$

$$^3P_{is} = \frac{P_{is}E_{isg}{}^2}{\mathrm{P}_{ig}(1-E_{igs})^2 + \mathrm{P}_{ir}E^2{}_{irg} + \mathrm{P}_{is}E^2{}_{isg}} \tag{3.7}$$

In general, these expressions can be written as

$$^jP_{ig} = \frac{P_{ig}(1-E_{igs})^{j-1}}{\mathrm{P}_{ig}(1-E_{igs})^{j-1} + \mathrm{P}_{ir}E_{irg}^{j-1} + \mathrm{P}_{is}E_{isg}^{j-1}} \tag{3.8}$$

$$^j\mathrm{P}_{ir} = \frac{\mathrm{P}_{ir}\mathrm{E}_{irg}^{j-1}}{\mathrm{P}_{ig}(1-\mathrm{E}_{igs})^{j-1} + \mathrm{P}_{ir}\mathrm{E}_{irg}^{j-1} + \mathrm{P}_{is}\mathrm{E}_{isg}^{j-1}} \tag{3.9}$$

$$^jP_{is} = \frac{P_{is}E_{isg}^{j-1}}{\mathrm{P}_{ig}(1-E_{igs})^{j-1} + \mathrm{P}_{ir}E_{irg}^{j-1} + \mathrm{P}_{is}E_{isg}^{j-1}} \tag{3.10}$$

Expressions for ^{j}PG, ^{j}PR and ^{j}PS

The probability of a component being good is

$$PG = \prod_{i=1}^{N} (1 - P_{ir} - P_{is}) \tag{3.11}$$

It is obvious that

$$^{1}PG = PG = \prod_{i=1}^{N} (1 - P_{ir} - P_{is})$$

$$^{1}PR = \sum_{i=1}^{N} P_{ir} - \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} P_{ir} P_{jr} + \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} \sum_{k=j+1}^{N} P_{ir} P_{jr} P_{kr} - \ldots\ldots\ldots \pm P_{1r} P_{2r} \ldots\ldots P_{Nr}$$

$$^{1}PS = 1 - {}^{1}PG - {}^{1}PR$$

The probability of a component being good entering the second cycle is

$$^{2}PG = \prod_{i=1}^{N} (1 - {}^{2}P_{ir} - {}^{2}P_{is})$$

Substituting the values of $^{2}P_{ir}$ and $^{2}P_{is}$ from equation (3) and (4) in the above equation gives

$$^{2}PG = \prod_{i=1}^{N} \left[\frac{P_{ig}(1 - E_{igs})}{P_{ig}(1 - E_{igs}) + P_{ir} E_{irg} + P_{is} E_{isg}} \right] \tag{3.12}$$

Similarly,

$$^{2}PR = \sum_{i=1}^{N} P_{ir} - \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} {}^{2}P_{ir} {}^{2}P_{jr} + \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} \sum_{k=j+1}^{N} {}^{2}P_{ir} {}^{2}P_{jr} {}^{2}P_{kr} - \ldots\ldots\ldots \pm {}^{2}P_{1r} {}^{2}P_{2r} \ldots\ldots {}^{2}P_{Nr} \tag{3.13}$$

and

$$^{2}PS = 1 - {}^{2}PG - {}^{2}PR \tag{3.14}$$

or in general, we can write

$$^{j}PG = \prod_{i=1}^{N} \left[\frac{P_{ig}(1 - E_{igs})^{j-1}}{P_{ig}(1 - E_{igs})^{j-1} + P_{ir} E_{irg}^{j-1} + P_{is} E_{isg}^{j-1}} \right] \tag{3.15}$$

$$ {}^{j}PR=\sum_{i=1}^{N} P_{ir} - \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} {}^{j}P_{ir}\,{}^{j}P_{jr} + \sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{k=j+1}^{N} {}^{j}P_{ir}\,{}^{j}P_{jr}\,{}^{j}P_{kr} - \ldots\ldots\pm {}^{j}P_{1r}\,{}^{j}P_{2r}\ldots\ldots{}^{j}P_{Nr} \tag{3.16} $$

and

$$ {}^{j}PS = 1 - {}^{j}PG - {}^{j}PR \tag{3.17} $$

When there is no inspection, the expected total cost per accepted unit will simply be the cost due to false acceptance of all defective components, i.e.,

$$ E(tc)|_{j=0} = C_a(1 - PG) \tag{3.18} $$

where C_a is the cost of false acceptance per accepted component and PG is given by equation (11).

The expected total cost per accepted component, after n cycles of inspection, can be written as

$$ E(tc)|_{j=n} = [TCFR + TCFA + TCI] / TA \tag{3.19} $$

where TCFR, TCFA, TCI and TA, defined in the notations, need to be determined.

To determine TCFR, TCFA, TCI and TA, analysis of different cycles of inspection is necessary.

Analysis of Cycle 1 of Inspection

All the components entering cycle 1 go to the first inspector, who inspects the first characteristic in each component in order to classify it as good, scrap or to be reworked. This is the first stage of inspection.

Stage 1.

The number of components entering into the first stage is

$$ M_{1,1} = M_1 \tag{3.20} $$

A component may have the following probabilities

$$PG_{1,1} = PG \tag{3.21}$$

$$PR_{1,1} = {}^{1}PR = \sum_{i=1}^{N} P_{ir} - \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} P_{ir}P_{jr} + \sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{k=j+1}^{N} P_{ir}P_{jr}P_{kr} - \ldots\ldots \pm P_{1r}P_{2r}\ldots\ldots P_{Nr}$$

$$PS_{1,1} = 1 - PG_{1,1} - PR_{1,1}$$

E(number of components falsely sent to rework) is

$$FR_{1,1} = M_{1,1}(PG_{1,1}E_{1gr} + PS_{1,1}E_{1sr}) \tag{3.22}$$

E(number of components falsely sent to scrap) is

$$FS_{1,1} = M_{1,1}(PG_{1,1}E_{1gs} + PR_{1,1}E_{1rs}) \tag{3.23}$$

E(number of falsely accepted components) is

$$FA_{1,1} = M_{1,1}[P_{1r}E_{1rg} + P_{1s}E_{1sg} + (1 - PG - P_{1r} - P_{1s})(1 - E_{1gr} - E_{1gs})] \tag{3.24}$$

E(number of correctly accepted components) is

$$CA_{1,1} = M_{1,1}PG_{1,1}(1 - E_{1gr} - E_{1gs}) \tag{3.25}$$

All accepted components in this stage go to the second inspector who inspects the second characteristic of each component. The following can be expected in the second stage of inspection.

Stage 2.

$$M_{2,1} = FA_{1,1} + CA_{1,1} + FGR_{1,1}$$

$$= M_{1,1}[P_{1r}E_{1rg} + P_{1s}E_{1sg} + (1 - PG - P_{1r} - P_{1s})(1 - E_{1gr} - E_{1gs}) + PG(1 - E_{1gr} - E_{1gs}) + PGE_{1gr}]$$

or

$$M_{2,1} = M_{1,1}[P_{1r}E_{1rg} + P_{1s}E_{1sg} + (1 - P_{1r} - P_{1s})(1 - E_{1gr} - E_{1gs}) + PGE_{1gr}] \tag{3.26}$$

$$PG_{2,1} = (1-{}^2P_{1r}-{}^2P_{1s})\prod_{i=2}^{N}(1-P_{ir}-P_{is}) \tag{3.27}$$

$$PR_{2,1} = {}^2P_{1r} + \sum_{i=2}^{N} P_{ir} - \sum_{i=2}^{N} {}^2P_{1r}P_{ir} + \sum_{i=2}^{N-1}\sum_{j=i+1}^{N} {}^2P_{ir}P_{jr} - \ldots\ldots\ldots\ldots\ldots\ldots\pm {}^2P_{1r}P_{2r}\ldots\ldots P_{Nr}$$

$$PS_{2,1} = 1 - PG_{2,1} - PR_{2,1}$$

$$FR_{2,1} = M_{2,1}(PG_{2,1}E_{2gr} + PS_{2,1}E_{2sr}) \tag{3.28}$$

$$FS_{2,1} = M_{2,1}(PG_{2,1}E_{2gs} + PR_{2,1}E_{2rs}) \tag{3.29}$$

$$FA_{2,1} = M_{2,1}[P_{2r}E_{2rg} + P_{2s}E_{2sg} + (1- PG_{2,1} - P_{2r} - P_{2s} + {}^2P_{1r}P_{2r} + {}^2P_{1s}P_{2s})(1-E_{2gr} - E_{2gs})] \tag{3.30}$$

$$CA_{2,1} = M_{2,1}PG_{2,1}(1 - E_{2gr} - E_{2gs}) \tag{3.31}$$

Similarly,

Stage 3.

$$M_{3,1} = M_{2,1}[P_{2r}E_{2rg} + P_{2s}E_{2sg} + (1 - P_{2r} - P_{2s} + P_{1r}P_{2r} + P_{1s}P_{2s})(1- E_{2gr} - E_{2gs}) + PG_{2,1}E_{2g} \tag{3.32}$$

$$PG_{2,1} = (1-{}^2P_{1r}-{}^2P_{1s})\,(1-{}^2P_{2r}-{}^2P_{2s})\prod_{i=3}^{N}(1-P_{ir}-P_{is}) \tag{3.33}$$

$$PR_{3,1} = {}^2P_{1r} + {}^2P_{2r} + \sum_{i=3}^{N} P_{ir} - {}^2P_{1r}\,{}^2P_{2r} - \sum_{i=2}^{N} {}^2P_{1r}\,{}^2P_{ir} + \sum_{i=3}^{N} {}^2P_{1r}\,{}^2P_{2r}P_{ir} - \ldots\ldots\pm {}^2P_{1r}\,{}^2P_{2r}\ldots\ldots P_{Nr}$$

$$PS_{3,1} = 1 - PG_{3,1} - PR_{3,1}$$

$$FR_{3,1} = M_{3,1}(PG_{3,1}E_{3gr} + M_{3,1}PS_{3,1}E_{3sr}) \tag{3.34}$$

$$FS_{3,1} = M_{1,1}PG_{1,1}(1 - E_{1gs})(1 - E_{2gs})E_{3gs} + M_{3,1}PR_{3,1}E_{3rs}$$

$$FA_{3,1} = M_{3,1}[P_{3r}E_{3rg} + P_{3s}E_{3sg} + (1 - PG_{3,1} - P_{3r} - P_{3s} + {}^2P_{1r}P_{3r} + {}^2P_{2r}P_{3r} - {}^2P_{1r}{}^2P_{2r}P_{3r}$$

$$+ {}^2P_{1s}P_{3s} + {}^2P_{2s}P_{3s} - {}^2P_{1s}{}^2P_{2s}P_{3s})(1 - E_{3gr} - E_{3gs})] \tag{3.35}$$

$$CA_{3,1} = M_{3,1}PG_{3,1}(1 - E_{1gs})(1 - E_{3gr} - E_{3gs})$$

So, we can generalize these expressions for the Nth stage.

Stage N.

$$M_{N,1} = M_{N-1,1}[P_{N-1r}E_{N-1r} + P_{N-1s}E_{N-1sg} + (1 - P_{N-1r} - P_{N-1s} + \sum_{k=1}^{N-2} P_{kr}P_{N-1,r} - \sum_{k=1}^{N-3}\sum_{l=k+1}^{N-2} P_{kr}P_{lr}P_{N-1,r}$$

$$+ \ldots \pm P_{1r}P_{2r}\ldots P_{N-1,r} + \sum_{k=1}^{N-2} P_{ks}P_{N-1s} + \sum_{k=1}^{N-3}\sum_{l=k+1}^{N-2} P_{ks}P_{ls}P_{N-1s} + \ldots \pm P_{1s}P_{2s}\ldots P_{N-1s})(1 - E_{N-1gr} -$$

$$E_{N-1gs}) + PG_{N-1,1}E_{N-1gr}] \tag{3.36}$$

$$PG_{2,1} = (1 - P_{Nr} - P_{Ns})\prod_{i=1}^{N-1}(1 - {}^2P_{ir} - {}^2P_{is})$$

$$PR_{N,1} = \sum_{i=1}^{N-1} {}^2P_{ir} + P_{Nr} - \sum_{i=1}^{N-1} {}^2P_{ir}P_{Nr} + \sum_{i=2}^{N-1} {}^2P_{1r}\,{}^2P_{ir}P_{ir} - \ldots\ldots \pm {}^2P_{1r}\,{}^2P_{2r}\ldots\ldots {}^2P_{N-1r}P_{Nr}$$

$$PS_{N,1} = 1 - PG_{N,1} - PR_{N,1}$$

$$FR_{N,1} = M_{N,1}(PG_{N,1}E_{Ngr} + M_{N,1}PS_{N,1}E_{Nsr}) \tag{3.37}$$

$$FS_{N,1} = M_{N,1}(PG_{N,1}E_{Ngs} + PR_{N,1}E_{Nrs}) \tag{3.38}$$

$$FA_{N,1} = M_{N,1}[P_{Nr}E_{Nrg} + P_{Ns}E_{Nsg} + (1 - PG_{N,1} - P_{Nr} - P_{Ns} + \sum_{k=1}^{N-1} P_{kr}P_{Nr} - \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} P_{kr}P_{lr}P_{Nr}$$

$$+\ldots\pm P_{1r}P_{2r}\ldots P_{N-1,r}+\sum_{k=1}^{N-1}P_{ks}P_{N-1s}+\sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1}P_{ks}P_{ls}P_{Ns}+\ldots\pm P_{1s}P_{2s}\ldots P_{N-1s})$$

$$(1-E_{Ngr}-E_{Ngs})] \tag{3.39}$$

$$CA_{N,1} = M_{N,1}PG_{N,1}(1-E_{Ngr}-E_{Ngs}) \tag{3.40}$$

This completes one cycle of inspection.

Results of Cycle 1

(1)E(number of accepted components) is

$$A(1) = FA_{N,1} + CA_{N,1} + FR_{N,1g}$$

$$= M_{N,1}[P_{Nr}E_{Nrg} + P_{Ns}E_{Nsg} + (1- PG_{N,1} - P_{Nr} - P_{Ns} + \sum_{k=1}^{N-1}P_{kr}P_{N-1,r} - \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1}P_{kr}P_{lr}P_{N-1,r}$$

$$+\ldots\pm P_{1r}P_{2r}\ldots P_{N-1,r}+\sum_{k=1}^{N-1}P_{ks}P_{N-1s}+\sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1}P_{ks}P_{ls}P_{N-1s}+\ldots\pm P_{1s}P_{2s}\ldots P_{N-1s})\text{x}$$

$$(1-E_{Ngr}-E_{Nsr}) + M_{N,1}PG_{N,1}(1 - E_{Ngr} - E_{Ngs}) + M_{N,1}PG_{N,1}E_{Ngr}$$

or

$$A(1) = M_{N,1}[P_{Nr}E_{Nrg} + P_{Ns}E_{Nsg} + (1- PG_{N,1} - P_{Nr} - P_{Ns} + \sum_{k=1}^{N-1}P_{kr}P_{N-1,r} - \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1}P_{kr}P_{lr}P_{N-1,r}$$

$$+\ldots\pm P_{1r}P_{2r}\ldots P_{N-1,r}+\sum_{k=1}^{N-1}P_{ks}P_{N-1s}+\sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1}P_{ks}P_{ls}P_{N-1s}+\ldots\pm P_{1s}P_{2s}\ldots P_{N-1s})\text{x}$$

$$(1-E_{Ngr}-E_{Nsr})] + M_{N,1}PG_{N,1}(1 - E_{Ngs}) \tag{3.41}$$

(2) Cost of false rejection

This is the cost associated with the components falsely sent to scrap.

$$CFR(1) = C_{fgs}\sum_{i=1}^{N} F_{igs} + C_{frs}\sum_{i=1}^{N} F_{irs}$$

where C_{fgs} = cost of falsely sending a good component to scrap.

C_{frs} = cost of falsely sending a reworkable component to scrap.

So,

$$CFR(1) = C_{fgs}\sum_{i=1}^{N}\left[M_{1,1}PG_{1,1}E_{igs}\prod_{k=0}^{i-1}(1 - E_{kgs}) \right] + C_{frs}\sum_{i=1}^{N}\left[M_{i,1}PR_{i,1}E_{irs}\right] \tag{3.42}$$

(3)Cost of false acceptance

This is the cost associated with the components classified as good falsely (i.e. they are either reworkable or are scrap) in the last stage of the cycle.

$$CFA(1) = C_aFA_{N,1}$$

where C_a is the cost of false acceptance per accepted component.

$$\begin{aligned} CFA(1) = C_a\, M_{N,1}[&P_{Nr}E_{Nrg} + P_{Ns}E_{Nsg} + (1- PG_{N,1} - P_{Nr} - P_{Ns} + \sum_{k=1}^{N-1} P_{kr}P_{Nr} - \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} P_{kr}P_{lr}P_{Nr} \\ &+ \dots \pm P_{1r}P_{2r}\dots P_{N-1,r} + \sum_{k=1}^{N-1} P_{ks}P_{N-1s} + \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} P_{ks}P_{ls}P_{Ns} + \dots \pm P_{1s}P_{2s}\dots P_{N-1s})\text{x} \\ &(1-E_{Ngr}-E_{Ngs})] \end{aligned} \tag{3.43}$$

(4)Cost of inspection

Cost of inspection is of two types.

C_{i1} = cost of inspecting a component at ith stage

C_{i2} = cost of inspecting a component at ith rework station. We assume that a very sensitive equipment (which is error-free) is installed there for inspection.

So,

$$C_{i2} > C_{i1}$$

$$CI_1(1) = \sum_{i=1}^{N} C_{i1} M_{i1}$$

or

$$CI_1(1) = C_{11}M_{11} + \sum_{i=2}^{N} C_{i1}\, M_{i-1,1}[P_{i-1r}E_{i-1rg} + P_{i-1s}E_{i-1sg} + (1 - P_{i-1r} - P_{i-1s})(1 - E_{i-1gr} - E_{i-1gs})$$

$$+ PG_{i-1,1}E_{i-1gr}] \tag{3.44}$$

$$CI_2(1) = \sum_{i=1}^{N} C_{i2}[\, M_{i,1}PG_{i,1}E_{igr} + M_{i,1}PS_{i,1}E_{isr} + M_{i,1}PR_{i,1}(1 - E_{irg} - E_{irs}] \tag{3.45}$$

So, the total cost of inspection for the cycle 1 is:

$$CI(1) = CI_1(1) + CI_2(1) \tag{3.46}$$

E (total cost per accepted component after one cycle of inspection) is

$E(tc)\,|_{j=1} = [CFR(1) + CFA(1) + CI(1)] / [A(1)]$,

where CFR(1), CFA(1), CI(1) and A(1) are given by (3.42), (3.43), (3.46) and (3.41), respectively.

For the sequencing of the characteristics, as in the Raouf et al [25], CFR(1), CFA(1) and A(1) will be unchanged irrespective of what sequence of characteristic inspection chosen. However, CI(1) is dependent on ordering of inspection. Therefore, to minimize $E(tc)\,|_{j=1}$, we must minimize CI(1).

Minimization of CI(1).

If the cost of inspection of each characteristic were the same, we would have inspected the characteristics with highest rejection rate first and lowest rejection rate last and the remaining sequenced in this order so as to minimize the cost of inspection.

And if the rejection rates for all the characteristics were the same, inspecting the characteristic having the least cost of inspection first, the one having the highest cost of inspection last, and remaining sequenced in this fashion, would result in minimizing the cost of inspection.

So, we would be inspecting first the characteristic with the least ratio of inspection cost, C_I, to the rejection rate, $R_{1,I}$, and last inspect the one with the highest ratio.

Rejection rate for any characteristic *i* is:

$$R_{i,1} = P_{is}(1 - E_{isg} - E_{isr}) + (1 - P_{is})((E_{igs} + E_{irs})$$

and for any characteristic *i*, the total cost of inspection is

$$C_i = C_{i1} + P_{ir}C_{i2}$$

Thus, we find the ratio $C_i/R_{i,1}$ for i = 1,2,....,N and inspect the characteristics in the ascending order of this ratio.

Analysis of Cycle 2 of Inspection

Number of components entering this cycle is $M_2 = A(1)$, where A(1) is given by equation (3.41).

Stage 1.

$M_{1,2} = M_2$

$PG_{1,2} = {}^{2}PG$

$PR_{1,2} = {}^{2}PR$

$PS_{1,2} = {}^{2}PS$

$FR_{1,1} = M_{1,2}(PG_{1,2}E_{1gr} + PS_{1,2}E_{1sr})$

$FS_{1,1} = M_{1,2}(PG_{1,2}E_{1gs} + PR_{1,2}E_{1rs})$

$FA_{1,1} = M_{1,2}[{}^{2}P_{1r}E_{1rg} + {}^{2}P_{1s}E_{1sg} + (1- {}^{2}PG - {}^{2}P_{1r} - {}^{2}P_{1s})(1-E_{1gr} - E_{1gs})]$

$CA_{1,1} = M_{1,1}PG_{1,1}(1 - E_{1gr} - E_{1gs})$

Stage 2.

$M_{2,2} = M_{1,2}[{}^{2}P_{1r}E_{1rg} + {}^{2}P_{1s}E_{1sg} + (1 - {}^{2}P_{1r} - {}^{2}P_{1s})(1-E_{1gr} - E_{1gs}) + PG_{1,2}E_{1gr}]$

$$PG_{2,2} = (1-{}^{3}P_{1r}-{}^{3}P_{1s})\prod_{i=2}^{N} (1-{}^{2}P_{ir}-{}^{2}P_{is})$$

$$PR_{2,2} = {}^{3}P_{1r} + \sum_{i=2}^{N} P_{ir} - \sum_{i=2}^{N} {}^{3}P_{1r}P_{ir} + \sum_{i=2}^{N-1}\sum_{j=i+1}^{N} {}^{3}P_{ir}\,{}^{2}P_{jr} - \ldots\ldots\pm{}^{3}P_{1r}\,{}^{2}P_{2r}\ldots\ldots{}^{2}P_{Nr}$$

$PS_{2,2} = 1 - PG_{2,2} - PR_{2,2}$

$FR_{2,2} = M_2\,(PG_{2,2}E_{2gr} + PS_{2,2}E_{2sr})$

$= M_2\,{}^{2}PG(1 - E_{1gs})E_{2gr} + M_{2,2}PS_{2,2}E_{2sr}$

$FS_{2,2} = M_{2,2}(PG_{2,2}E_{2gs} + PR_{2,2}E_{2rs})$

$$= M_2{}^2PG(1 - E_{1gs})E_{2gs} + M_{2,1}PR_{2,1}E_{2rs}$$

$$FA_{2,2} = M_{2,2}[{}^2P_{2r}E_{2rg} + {}^2P_{2s}E_{2sg} + (1- PG_{2,2} - {}^2P_{2r} - {}^2P_{2s} + {}^3P_{1r}\,{}^2P_{2r} + {}^3P_{1s}\,{}^2P_{2s})(1-E_{2gr} - E_{2gs})]$$

$$CA_{2,2} = M_2\,{}^2PG(1 - E_{1gs})\,(1 - E_{2gr} - E_{2gs})$$

Stage N.

$$M_{N,2} = M_{N-1,2}[{}^2P_{N-1r}E_{N-1r} + {}^2P_{N-1s}E_{N-1sg} + (1 - {}^2P_{N-1r} - {}^2P_{N-1s} + \sum_{k=1}^{N-2} {}^2P_{kr}\,{}^2P_{N-1,r} -$$

$$\sum_{k=1}^{N-3}\sum_{l=k+1}^{N-2} {}^2P_{kr}\,{}^2P_{lr}\,{}^2P_{N-1,r} + \ldots \pm {}^2P_{1r}\,{}^2P_{2r}\ldots{}^2P_{N-1,r} + \sum_{k=1}^{N-3}\sum_{l=k+1}^{N-2} {}^2P_{ks}\,{}^2P_{ls}\,{}^2P_{N-1s} + \ldots$$

$$\pm {}^2P_{1s}\,{}^2P_{2s}\ldots{}^2P_{N-1s})(1 - E_{N-1gr} - E_{N-1gs}) + PG_{N-1,2}E_{N-1gr}]$$

$$PG_{N,2} = (1-{}^2P_{Nr}-{}^2P_{Ns})\prod_{i=1}^{N-1} (1-{}^3P_{ir}-{}^3P_{is})$$

$$PR_{N,2} = \sum_{i=1}^{N-1} {}^3P_{ir} + {}^2P_{Nr} - \sum_{i=1}^{N-1} {}^3P_{ir}\,{}^2P_{Nr} + \sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1} {}^3P_{ir}\,{}^3P_{jr} - \ldots\ldots \pm {}^3P_{1r}\,{}^3P_{2r}\ldots\ldots{}^3P_{N-1r}P_{Nr}$$

$$PS_{N,2} = 1- PG_{N,2} - PR_{N,2}$$

$$FR_{N,2} = M_{N,2}(PG_{N,2}E_{Ngr} + M_{N,2}PS_{N,2}E_{Nsr})$$

$$FS_{N,2} = M_{N,2}(PG_{N,2}E_{Ngs} + M_{N,2}PR_{N,2}E_{Nrs})$$

$$FA_{N,2} = M_{N,2}[{}^2P_{Nr}E_{N-1r} + {}^2P_{Ns}E_{Nsg} + (1 - PG_{N,2} - {}^2P_{Nr} - {}^2P_{Ns} + \sum_{k=1}^{N-1} {}^2P_{kr}\,{}^2P_{N,r} -$$

$$\sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^2P_{kr}\,{}^2P_{lr}\,{}^2P_{Nr} + \ldots \pm {}^2P_{1r}\,{}^2P_{2r}\ldots{}^2P_{N,r} + \sum_{k=1}^{N-1} {}^2P_{ks}\,{}^2P_{Ns} + \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^2P_{ks}\,{}^2P_{ls}\,{}^2P_{Ns}$$

$$+\ldots \pm {}^2P_{1s}\,{}^2P_{2s}\ldots{}^2P_{Ns})(1 - E_{Ngr} - E_{Ngs}) + PG_{N,2}E_{Ngr}]$$

$$CA_{N,2} = M_{N,2}\,PG_{N,2}(1 - E_{Ngr} - E_{Ngs})$$

Results of Cycle 2

(1)Expected number of accepted components

$$A(2) = M_{N,2}[{}^2P_{Nr}E_{N\text{-}1r} + {}^2P_{Ns}E_{Nsg} + (1 - PG_{N,2} - {}^2P_{Nr} - {}^2P_{Ns} + \sum_{k=1}^{N-1} {}^2P_{kr}\,{}^2P_{N,r} -$$

$$\sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^2P_{kr}\,{}^2P_{lr}\,{}^2P_{Nr} + \ldots \pm {}^2P_{1r}\,{}^2P_{2r}\ldots{}^2P_{N,r} + \sum_{k=1}^{N-1} {}^2P_{ks}\,{}^2P_{Ns} + \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^2P_{ks}\,{}^2P_{ls}\,{}^2P_{Ns}$$

$$+\ldots \pm {}^2P_{1s}\,{}^2P_{2s}\ldots{}^2P_{Ns})(1 - E_{Ngr} - E_{Ngs})] + M_{N,2}\,PG_{N,2}(1 - E_{Ngs})$$

(2)Cost of false rejection

$$CFR(2) = C_{fgs}\sum_{i=1}^{N}\left[M_{i,2}PG_{i,2}E_{igs}\right] + C_{frs}\sum_{i=1}^{N}\left[M_{i,2}PR_{i,2}E_{irs}\right]$$

(3)Cost of false acceptance

$$CFA(2) = C_a\, M_{N,2}[{}^2P_{Nr}E_{N\text{-}1r} + {}^2P_{Ns}E_{Nsg} + (1 - PG_{N2} - {}^2P_{Nr} - {}^2P_{Ns} + \sum_{k=1}^{N-1} {}^2P_{kr}\,{}^2P_{N,r}$$

$$-\sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^2P_{kr}\,{}^2P_{lr}\,{}^2P_{Nr} + \ldots \pm {}^2P_{1r}\,{}^2P_{2r}\ldots{}^2P_{N,r} + \sum_{k=1}^{N-1} {}^2P_{ks}\,{}^2P_{Ns} +$$

$$\sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^2P_{ks}\,{}^2P_{ls}\,{}^2P_{Ns} + \ldots \pm {}^2P_{1s}\,{}^2P_{2s}\ldots{}^2P_{Ns})(1 - E_{Ngr} - E_{Ngs})$$

(4)Cost of inspection

$$CI_1(2) = C_{11}M_{12} + \sum_{i=2}^{N} C_{i1}\, M_{i\text{-}1,2}[{}^2P_{i\text{-}1r}E_{i\text{-}1rg} + {}^2P_{i\text{-}1s}E_{i\text{-}1sg} + (1 - {}^2P_{i\text{-}1r} - {}^2P_{i\text{-}1s})(1 - E_{i\text{-}1gr} - E_{i\text{-}1gs})$$

$+ PG_{i-1,2}E_{i-1gr}]$

$$CI_2(2) = \sum_{i=1}^{N} C_{i2}[\ M_{i,2}\ PG_{i,2}E_{igr} + M_{i,2}PS_{i,2}E_{isr} + M_{i,2}PR_{i,2}(1 - E_{irg} - E_{irs})\ ]$$

So, the total cost of inspection for the cycle 2 is:

$$CI(2) = CI_1(2) + CI_2(2)$$

The ratio in order to determine the optimal ordering of characteristics in the second cycle is

$$C_i / R_{i,2} \ \text{ for } i = 1,2,\ldots,N.$$

where

$$R_{i,2} = {}^2P_{is}(1 - E_{isg} - E_{isr}) + (1 - {}^2P_{is})(\ (E_{igs} + E_{irs})$$

In a similar fashion we can write the results of the n^{th} cycle.

Results of Cycle n

(1) Expected number of accepted components

$$A(n) = M_{N,n}[{}^nP_{Nr}E_{Nrg} + {}^nP_{Ns}E_{Nsg} + (1 - PG_{Nn} - {}^nP_{Nr} - {}^nP_{Ns} + \sum_{k=1}^{N-1} {}^nP_{kr}\,{}^nP_{Nr} -$$

$$\sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^nP_{kr}\,{}^nP_{lr}\,{}^nP_{Nr} + \ldots \pm {}^nP_{1r}\,{}^nP_{2r}\ldots{}^nP_{N,r} + \sum_{k=1}^{N-1} {}^nP_{ks}\,{}^nP_{Ns} + \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^nP_{ks}\,{}^nP_{ls}\,{}^nP_{Ns} +$$

$$\ldots \pm {}^nP_{1s}\,{}^nP_{2s}\ldots{}^nP_{Ns})(1 - E_{Ngr} - E_{Ngs})] + M_n\,{}^nPG(1 - E_{Ngs})\left[\prod_{k=0}^{N-1}(1 - E_{kgs})\right]\ ;\ E_{0gs} = 0$$

(3.47)

(2)Cost of false rejection

$$CFR(n) = C_{fgs}\sum_{i=1}^{N}\left[M_n{}^nPGE_{igs}\prod_{k=0}^{i-1}(1-E_{kgs})\right] + C_{frs}\sum_{i=1}^{N}\left[M_{i,n}PR_{i,n}E_{irs}\right] \tag{3.48}$$

(3)Cost of false acceptance

$$CFA(n) = C_a\, M_{N,n}[{}^nP_{Nr}E_{Nrg} + {}^nP_{Ns}E_{Nsg} + (1 - PG_{Nn} - {}^nP_{Nr} - {}^nP_{Ns} + \sum_{k=1}^{N-1}{}^nP_{kr}{}^nP_{Nr} -$$

$$\sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1}{}^nP_{kr}{}^nP_{lr}{}^nP_{Nr} + \ldots \pm{}^nP_{1r}{}^nP_{2r}\ldots{}^nP_{N,r} + \sum_{k=1}^{N-1}{}^nP_{ks}{}^nP_{Ns} + \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1}{}^nP_{ks}{}^nP_{ls}{}^nP_{Ns}$$

$$+ \ldots \pm{}^nP_{1s}{}^nP_{2s}\ldots{}^nP_{Ns})(1 - E_{Ngr} - E_{Ngs})] \tag{3.49}$$

(4)Cost of inspection

$$CI_1(n) = C_{11}M_{1n} + \sum_{i=2}^{N}C_{i1}\, M_{i-1,n}[{}^nP_{i-1r}E_{i-1rg} + {}^nP_{i-1s}E_{i-1sg} + (1 - {}^nP_{i-1r} - {}^nP_{i-1s})(1 - E_{i-1gr} - E_{i-1gs})$$

$$+ PG_{i-1,n}E_{i-1gr}]$$

$$CI_2(n) = \sum_{i=1}^{N}C_{i2}\left[M_n{}^nPG\left[\prod_{k=0}^{i-1}(1-E_{kgs})\right]E_{igr} + M_{i,n}PS_{i,n}E_{isr} + M_{i,n}PR_{i,n}(1 - E_{irg} - E_{irs})\right]$$

So, the total cost of inspection for the cycle n is:

$$CI(n) = CI_1(n) + CI_2(n) \tag{3.50}$$

The ratio in order to determine the optimal ordering of characteristics in the second cycle is

$$C_i / R_{i,n} \text{ for } i = 1,2,\ldots,N.$$

where

$$R_{i,n} = {}^{n}P_{is}(1 - E_{isg} - E_{isr}) + (1 - {}^{n}P_{is})(\,(E_{igs} + E_{irs})$$

Next, we determine the general expressions for the components of the total cost i.e. total cost of false acceptance TCFA, total cost of false rejection TCFR, total cost of inspection TCI and total number of accepted components TA.

Determination of TCFR.

We can write as

$$TCFR = \sum_{j=1}^{n}[CFR(j)]$$

$$= \sum_{j=1}^{n}\{\ C_{fgs}\sum_{i=1}^{N}\left[M_j\,{}^{j}PGE_{igs}\prod_{k=0}^{i-1}(1 - E_{kgs})\ \right] + C_{frs}\sum_{i=1}^{N}\left[M_{i,j}PR_{i,j}E_{irs}\ \right]\};E_{0gs}=0 \quad (3.51)$$

Determination of TCFA.

It is obvious that the total number of accepted components in the inspection are those that are accepted in the nth cycle. So,

$$TCFA = CFA(n)$$

$$= C_a\, M_{N,n}[{}^nP_{Nr}E_{N-1r} + {}^nP_{Ns}E_{Nsg} + (1 - {}^nP_{Nr} - {}^nP_{Ns} + \sum_{k=1}^{N-1} {}^nP_{kr}\,{}^nP_{Nr} - \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^nP_{kr}\,{}^nP_{lr}\,{}^nP_{Nr} +$$

$$\ldots \pm {}^nP_{1r}\,{}^nP_{2r}\ldots{}^nP_{N,r} + \sum_{k=1}^{N-1} {}^nP_{ks}\,{}^nP_{Ns} + \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^nP_{ks}\,{}^nP_{ls}\,{}^nP_{Ns} + \ldots \pm {}^nP_{1s}\,{}^nP_{2s}\ldots{}^nP_{Ns})x$$

$$(1 - E_{Ngr} - E_{Ngs}) \tag{3.52}$$

Determination of TCI

The total inspection cost is the sum of the components in each cycle. Thus,

$$TCI = \sum_{j=1}^{n}[CI1(j) + CI2(j)]$$

$$= \sum_{j=1}^{n}\{\ C_{11}M_{1n} + \sum_{i=2}^{N} C_{i1}\ M_{i-1,n}[{}^nP_{i-1r}E_{i-1rg} + {}^nP_{i-1s}E_{i-1sg} + (1 - {}^nP_{i-1r} - {}^nP_{i-1s})(1 - E_{i-1gr} - E_{i-1gs})$$

$$+PG_{i-1,n}E_{i-1gr}]\ \} + \sum_{j=1}^{n}\ \{\sum_{i=1}^{N} C_{i2}[\ M_n\,{}^nPG\left[\prod_{k=0}^{i-1}(1 - E_{kgs})\right]E_{igr} + M_{i,n}PR_{i,n}E_{isr}\ ]\ \} \tag{3.53}$$

Determination of TA

Again, the total number of accepted components will be those which are accepted in the nth cycle. So,

$$TA\ = A(n)$$

$$= M_{N,n}[{}^nP_{Nr}E_{N\text{-}1r} + {}^nP_{Ns}E_{Nsg} + (1 - {}^nP_{Nr} - {}^nP_{Ns} + \sum_{k=1}^{N-1} {}^nP_{kr}\,{}^nP_{Nr} - \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^nP_{kr}\,{}^nP_{lr}\,{}^nP_{Nr} + \ldots$$

$$\pm {}^nP_{1r}\,{}^nP_{2r}\ldots{}^nP_{N,r} + \sum_{k=1}^{N-1} {}^nP_{ks}\,{}^nP_{Ns} + \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^nP_{ks}\,{}^nP_{ls}\,{}^nP_{Ns} + \ldots \pm {}^nP_{1s}\,{}^nP_{2s}\ldots{}^nP_{Ns})$$

$$(1 - E_{Ngr} - E_{Ngs})] + M_n\,{}^nPG(1 - E_{Ngs})\left[\prod_{k=0}^{N-1}(1 - E_{kgs})\right];\ E_{0gs} = 0 \tag{3.54}$$

Substituting the values of TCFR, TCFA, TCI and TA from (3.51), (3.52), (3.53) and (3.54) into equation (3.19) gives the expected total cost per accepted component for n cycles of inspection, i.e. $E(tc)|_{j=0}$, where ${}^jP_{ir}$, ${}^jP_{is}$ and ${}^jPG$ are given by (3.9), (3.10) and (3.15) respectively. The expressions for $PG_{I,j}$, M_j and $M_{i,j}$ are as follows:

$$PG_{i,j} = \begin{cases} {}^jPG; i = 1 \\ PG_{i-1,j}(1 - E_{i-1gs})/[{}^jP_{i-1r}E_{i-1rg} + {}^jP_{i-1s}E_{i-1sg} + (1 - {}^jP_{i-1r} - {}^jP_{i-1s} + \sum_{k=1}^{i-2} {}^jP_{kr}\,{}^jP_{i-1r} \\ -\sum_{k=1}^{i-3}\sum_{l=k+1}^{i-2} {}^jP_{kr}\,{}^jP_{lr}\,{}^jP_{i-1,r} + \ldots \pm {}^jP_{1r}\,{}^jP_{2r}\ldots{}^jP_{i-1r} + \sum_{k=1}^{i-2} {}^jP_{ks}\,{}^jP_{i-1s} \\ -\sum_{k=1}^{i-3}\sum_{l=k+1}^{i-2} {}^jP_{ks}\,{}^jP_{ls}\,{}^jP_{i-1s} + \ldots \pm {}^jP_{1s}\,{}^jP_{2s}\ldots{}^jP_{i-1s}\ \big)(1 - E_{i-1gr} - E_{i-1gs}) + PG_{i-1,j}E_{i-1gr}\ \big\}\ i = 2.., N \end{cases} \tag{3.55}$$

$$PR_{i,j} = \begin{cases} {}^jPR; i = 1 \\ PR_{i-1,j}(1 - E_{i-1gs})/[{}^jP_{i-1r}E_{i-1rg} + {}^jP_{i-1s}E_{i-1sg} + (1 - {}^jP_{i-1r} - {}^jP_{i-1s} + \sum_{k=1}^{i-2} {}^jP_{kr}\,{}^jP_{i-1r} \\ -\sum_{k=1}^{i-3}\sum_{l=k+1}^{i-2} {}^jP_{kr}\,{}^jP_{lr}\,{}^jP_{i-1,r} + \ldots \pm {}^jP_{1r}\,{}^jP_{2r}\ldots{}^jP_{i-1r} + \sum_{k=1}^{i-2} {}^jP_{ks}\,{}^jP_{i-1s} \\ -\sum_{k=1}^{i-3}\sum_{l=k+1}^{i-2} {}^jP_{ks}\,{}^jP_{ls}\,{}^jP_{i-1s} + \ldots \pm {}^jP_{1s}\,{}^jP_{2s}\ldots{}^jP_{i-1s}\ \big)(1 - E_{i-1gr} - E_{i-1gs}) + PG_{i-1,j}E_{i-1gr}\ \big\}\ i = 2.., N \end{cases} \tag{3.56}$$

$$PS_{i,j} = 1 - PG_{i,j} - PR_{i,j} \tag{3.57}$$

$$M_{i,j} = \begin{cases} M_j; i=1 \\ M_{i-1,j}[{}^jP_{i-1r}E_{i-1rg} + {}^jP_{i-1s}E_{i-1sg} + (1 - {}^jP_{i-1r} - {}^jP_{i-1s} + \sum_{k=1}^{i-2} {}^jP_{kr}\,{}^jP_{i-1r} \\ -\sum_{k=1}^{i-3}\sum_{l=k+1}^{i-2} {}^jP_{kr}\,{}^jP_{lr}\,{}^jP_{i-1,r} + \ldots \pm {}^jP_{1r}\,{}^jP_{2r} \ldots {}^jP_{i-1r} + \sum_{k=1}^{i-2} {}^jP_{ks}\,{}^jP_{i-1s} \\ -\sum_{k=1}^{i-3}\sum_{l=k+1}^{i-2} {}^jP_{ks}\,{}^jP_{ls}\,{}^jP_{i-1s} + \ldots \pm {}^jP_{1s}\,{}^jP_{2s} \ldots {}^jP_{i-1s}\ \big)\big(1 - E_{i-1gr} - E_{i-1gs}\big) + PG_{i-1,j}E_{i-1gr}\ \big]; i = 2,..N \end{cases} \tag{3.58}$$

$$M_j = \begin{cases} M_1;\ j=1 \\ M_{N,j-1}[{}^{j-1}P_{Nr}E_{Nrg} + {}^{j-1}P_{Ns}E_{Nsg} + (1 - PG_{N,j-1} - {}^{j-1}P_{Nr} - {}^{j-1}P_{Ns} + \sum_{k=1}^{N-1} {}^{j-1}P_{kr}\,{}^{j-1}P_{Nr} \\ -\sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^{j-1}P_{kr}\,{}^{j-1}P_{lr}\,{}^{j-1}P_{Nr} + \ldots \pm {}^{j-1}P_{1r}\,{}^{j-1}P_{2r} \ldots {}^{j-1}P_{Nr} + \sum_{k=1}^{N-1} {}^{j-1}P_{ks}\,{}^{j-1}P_{Ns} \\ -\sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} {}^{j-1}P_{ks}\,{}^{j-1}P_{ls}\,{}^{j-1}P_{Ns} + \ldots \pm {}^{j-1}P_{1s}\,{}^{j-1}P_{2s} \ldots {}^{j-1}P_{Ns}\ \big)\big(1 - E_{Ngr} - E_{Ngs}\big)] \\ \\ + M_{j-1}\ {}^{j-1}PG(1 - E_{Ngs})\left[\prod_{k=0}^{N-1}(1 - E_{kgs})\right]; E_{0gs} = 0;\ j = 2,3,\ldots,n. \end{cases} \tag{3.59}$$

The ratio to determine the optimal sequence of characteristics for each cycle of inspection is $C_i/R_{i,j}$ for $i = 1,2,\ldots,N$, where

$$C_i = C_{i1} + {}^jP_{ir}C_{i2}$$

and

$$R_{i,j} = {}^jP_{is}(1 - E_{isg} - E_{isr}) + (1 - {}^jP_{is})((E_{igs} + E_{irs})\ ;\ j = 1,2,..,n \tag{3.60}$$

3.4 Computational Procedure

With the help of the incoming data, i.e. the probabilities of the rework, scrap and the errors of misclassification, we would follow the following computational procedure to find the optimal number of inspection cycle.

Step 1. Compute PG and $E(tc)|_{j=0}$ using (3.11) and (3.18) respectively.

Step 2. Set $j = j + 1$. Compute ${}^{j}P_{ir}$, ${}^{j}P_{is}$ using (3.9) and (3.10). Sequence the characteristics according to the ratio $C_i/R_{i,j}$, using (3.60).

Step 3. Compute ${}^{j}PG$, ${}^{j}PR$, ${}^{j}PS$, $PG_{i,j}$, $PR_{i,j}$, $PS_{i,j}$, $M_{i,j}$ and M_j using (3.15), (3.16), (3.17), (3.55), (3.56) ,(3.57), (3.58) and (3.59) respectively.

Step 4. Compute A(j), CFR(j), CFA(j), and CI(j) using (3.47), (3.48), (3.49) and (3.50) respectively.

Step 5. Compute TCFR, TCFA, TCI and TA using (3.51), (3.52), (3.53) and (3.54) respectively.

Step 6. Compute $E(tc)|_j$ using (3.19).

Step 7. If $E(tc)|_j < E(tc)|_{j-1}$, go to step 2. Otherwise stop.

This procedure provides the optimal ordering of characteristics for each cycle of inspection and gives the optimal number of inspection i.e. $n = j-1$. The given set of data includes P_{ig}, P_{ir}, P_{is}, E_{igs}, E_{igr}, E_{irg}, E_{irs}, E_{isg}, E_{isr} for $i = 1,2,..,N$. The estimates of costs i.e. C_a, C_{fgs}, C_{frs}, $C_{i1}$and C_{i2} are also assumed to be given.

3.5 Illustrative Example

In order to illustrate the model presented in this chapter, the following example is provided. A program is developed implementing the algorithm in section (3.4) and is given in Appendix (A). It is used to obtain the optimal number of repeat inspection.

Example:

Assuming that the following data is given

$N = 3$, $M = 100$, $C_a = 100000$, $C_{i1} = 100$, $C_{i2} = 5000$, $C_{fgs} = 10000$, $C_{frs} = 5000$

$P_{ir} = 0.1$, 0.05, and 0.05 respectively, and $P_{is} = 0.1$, 0.05, and 0.15 respectively

$E_{gs} = 0.03$, $E_{sg} = 0.05$, $E_{gr} = E_{rg} = E_{sr} = E_{rs} = 0.05$

Solving this example using the proposed model gives the following results:

Expected Total Cost per accepted component without inspection = 42400.00

__________ Cycle 1 __________

PG(1) 0.5760000

A(1) 51

Etc(1) 9167.7320

__________ Cycle 2 __________

PG(2) 0.9691657

A(2) 44

Etc(2) 9267.9400

Optimal no. of Inspection = 1

Thus the optimal number of inspection cycles is 1 when the characteristics defective rates are independent with the given data.

3.6 Effect of Sequencing

To investigate the effect of sequencing we compare the results of the model for two cases viz (i) using the optimal sequence of characteristics for each cycle of inspection and (ii) using a fixed sequence of characteristics for each cycle of inspection The fixed sequence is the optimal sequence in the first cycle. The comparison is carried out using randomly generated inspection problems. The results of the comparison indicate that the inspection model slightly better in terms of cost in the case of using the optimal sequence as shown in Table 3.1.

3.6.1 Random Problem Generation

The parameters of the problems generated are C_a, C_{i1}, C_{i2}, C_{fgs}, C_{frs}, P_{ir}, P_{is},e_{gs}, e_{sg} and the other errors of misclassification. The costs are all assumed to be uniformly distributed. While the errors and the probabilities of rework and scrap are taken to be normally distributed with known mean μ and variance σ^2. The technique used to generate random variables is obtained from [18]. Ca is assumed to vary between 100000 and 1000000. Cost of inspection and that of misclassification are taken to be between 10 – 100 and 500 –1000 respectively. The mean and variance for the probabilities and the errors are taken to be 0.1, 0.03 and 0.05, 0.03 respectively. Using these assumptions, a computer program is developed, given in Appendix (C), to generate problems for the inspection model.

3.6.2 Results of Comparison

The results of solving hundred randomly generated problems for the two cases viz the optimal sequence and the fixed sequence are summarized in Table 3.1. The criterion of the comparison is taken to be the expected total cost of inspection.

Table 3.1 Effect of Sequencing

	Optimal Sequence					Fixed Sequence					%age differ.
S.No	AOQ	A(n)	ATI	ETC	n	AOQ	A(n)	ATI	ETC	n	in cost
1	0.0035	17	510	72033	2	0.0035	17	510	72033	2	0.00
2	0.0105	12	485	88797	2	0.0105	12	485	88797	2	0.00
3	0.0006	28	846	87120	3	0.0006	28	846	87601	3	0.55
4	0.0054	8	438	82108	2	0.0054	8	438	82145	2	0.04
5	0.0147	16	500	101919	2	0.0147	16	500	101919	2	0.00
6	0.0128	12	502	130904	2	0.0128	12	502	130904	2	0.00
7	0.0013	11	644	84405	3	0.0013	11	644	84557	3	0.18
8	0.0015	21	791	66390	3	0.0015	21	791	66390	3	0.00
9	0.0037	22	566	110017	2	0.0037	22	566	110017	2	0.00
10	0.0010	17	718	59306	3	0.0010	17	718	59306	3	0.00
11	0.0009	22	784	79368	3	0.0009	22	784	79419	3	0.06
12	0.0006	30	875	78601	3	0.0006	30	875	78601	3	0.00
13	0.0042	15	510	72371	2	0.0042	15	510	72392	2	0.03
14	0.0015	8	565	66782	3	0.0015	8	565	66782	3	0.00
15	0.0043	10	442	85792	2	0.0043	10	442	85792	2	0.00
16	0.0051	26	611	94997	2	0.0051	26	611	95024	2	0.03
17	0.0056	14	487	94304	2	0.0056	14	487	94316	2	0.01
18	0.0099	13	489	73196	2	0.0099	13	489	73250	2	0.07
19	0.0015	9	602	78517	3	0.0015	9	602	78517	3	0.00
20	0.0007	22	799	93824	3	0.0007	22	799	93853	3	0.03
21	0.0008	24	795	73005	3	0.0008	24	795	73011	3	0.01
22	0.0054	20	572	83357	2	0.0054	20	571	83387	2	0.04
23	0.0064	15	507	139929	2	0.0064	15	507	139929	2	0.00
24	0.0037	17	538	111055	2	0.0037	17	538	111136	2	0.07
25	0.0081	22	602	131340	2	0.0081	22	602	131340	2	0.00
26	0.0093	11	494	109459	2	0.0093	11	493	109474	2	0.01
27	0.0070	10	453	49499	2	0.0070	10	453	49499	2	0.00
28	0.0115	11	489	56577	2	0.0115	11	489	56577	2	0.00
29	0.0074	17	540	73876	2	0.0074	17	540	73876	2	0.00
30	0.0014	19	707	72588	3	0.0014	19	707	72588	3	0.00
31	0.0020	19	737	84401	3	0.0020	19	737	84401	3	0.00
32	0.0117	21	575	117581	2	0.0117	21	575	117581	2	0.00
33	0.0067	24	544	103218	2	0.0067	24	544	103218	2	0.00
34	0.0117	9	449	93769	2	0.0117	9	449	93769	2	0.00
35	0.0059	17	525	126181	2	0.0059	17	525	126181	2	0.00
36	0.0014	7	593	98398	3	0.0014	7	593	98398	3	0.00
37	0.0009	13	685	109437	3	0.0009	13	687	109982	3	0.50
38	0.0006	15	667	72506	3	0.0006	15	666	72593	3	0.12

Table 3.1 Effect of Sequencing (Contd.)

39	0.0009	8	552	88144	3	0.0009	8	552	88274	3	0.15
40	0.0063	12	480	42026	2	0.0063	12	480	42036	2	0.02
41	0.0007	14	697	74051	3	0.0007	14	697	74051	3	0.00
42	0.0064	22	580	81609	2	0.0064	22	580	81644	2	0.04
43	0.0023	26	584	154484	2	0.0023	26	585	154492	2	0.01
44	0.0012	16	744	90063	3	0.0012	16	744	90063	3	0.00
45	0.0026	18	519	65901	2	0.0026	18	519	65901	2	0.00
46	0.0134	11	458	142481	2	0.0134	11	457	142710	2	0.16
47	0.0047	33	657	118319	2	0.0047	33	657	118432	2	0.10
48	0.0115	9	455	134423	2	0.0115	9	455	134507	2	0.06
49	0.0013	13	653	102272	3	0.0013	13	653	102288	3	0.02
50	0.0092	12	488	79039	2	0.0092	12	488	79039	2	0.00
51	0.0013	16	712	88741	3	0.0013	16	710	89072	3	0.37
52	0.0011	17	725	68805	3	0.0011	18	725	68819	3	0.02
53	0.0050	22	547	96672	2	0.0050	22	547	96784	2	0.12
54	0.0079	18	540	97787	2	0.0079	18	540	97787	2	0.00
55	0.0012	22	813	70346	3	0.0012	22	813	70433	3	0.12
56	0.0004	12	653	73707	3	0.0004	12	653	73773	3	0.09
57	0.0073	17	553	102033	2	0.0073	17	553	102068	2	0.03
58	0.0015	11	602	75709	3	0.0015	11	602	75805	3	0.13
59	0.0092	11	466	129452	2	0.0092	11	466	129452	2	0.00
60	0.0128	12	508	97585	2	0.0128	12	508	97585	2	0.00
61	0.0013	18	713	103492	3	0.0013	18	714	103635	3	0.14
62	0.0007	16	691	40255	3	0.0007	16	691	40334	3	0.20
63	0.0050	18	550	108136	2	0.0050	18	550	108136	2	0.00
64	0.0014	19	729	103939	3	0.0014	19	729	104103	3	0.16
65	0.0056	22	570	97455	2	0.0056	22	570	97455	2	0.00
66	0.0005	27	824	52834	3	0.0005	27	824	52834	3	0.00
67	0.0060	13	479	37487	2	0.0060	13	479	37493	2	0.02
68	0.0008	12	656	82348	3	0.0008	12	656	82348	3	0.00
69	0.0092	12	460	82116	2	0.0092	12	460	82134	2	0.02
70	0.0009	13	642	67794	3	0.0009	13	642	67794	3	0.00
71	0.0013	21	790	62105	3	0.0013	21	790	62314	3	0.34
72	0.0004	13	622	66999	3	0.0004	13	621	67105	3	0.16
73	0.0036	21	563	44796	2	0.0036	21	563	44857	2	0.13
74	0.0008	10	611	71727	3	0.0008	10	611	71727	3	0.00
75	0.0109	15	513	98429	2	0.0109	15	513	98429	2	0.00
76	0.0004	13	637	33613	3	0.0004	13	639	33704	3	0.27
77	0.0027	19	539	83821	2	0.0027	19	539	83821	2	0.00
78	0.0070	15	507	93057	2	0.0070	15	507	93103	2	0.05
79	0.0036	19	565	58234	2	0.0036	19	565	58314	2	0.14
80	0.0084	13	508	83029	2	0.0084	13	508	83070	2	0.05

Table 3.1 Effect of Sequencing (Contd.)

81	0.0002	25	802	76486	3	0.0002	25	802	76535	3	0.06
82	0.0025	28	598	68127	2	0.0025	28	598	68127	2	0.00
83	0.0019	9	631	104182	3	0.0019	9	631	104258	3	0.07
84	0.0007	19	759	111920	3	0.0007	19	760	112210	3	0.26
85	0.0079	12	486	122905	2	0.0079	12	486	122940	2	0.03
86	0.0090	14	525	70951	2	0.0090	14	525	70951	2	0.00
87	0.0044	23	568	102852	2	0.0044	23	568	102852	2	0.00
88	0.0145	14	521	180402	2	0.0145	14	519	181305	2	0.50
89	0.0005	13	637	62097	3	0.0005	13	637	62097	3	0.00
90	0.0035	13	468	99584	2	0.0035	13	468	99625	2	0.04
91	0.0041	18	542	120692	2	0.0041	18	542	120692	2	0.00
92	0.0021	10	589	74880	3	0.0021	10	589	74880	3	0.00
93	0.0021	12	674	49571	3	0.0021	12	675	49763	3	0.39
94	0.0075	13	496	53051	2	0.0075	13	496	53051	2	0.00
95	0.0019	18	749	94366	3	0.0019	18	749	94366	3	0.00
96	0.0052	16	516	99490	2	0.0052	16	516	99490	2	0.00
97	0.0013	17	692	79077	3	0.0013	17	692	79077	3	0.00
98	0.0102	12	498	74904	2	0.0102	12	498	74904	2	0.00
99	0.0109	15	513	98429	2	0.0109	15	513	98429	2	0.00
100	0.0015	8	565	66782	3	0.0015	8	565	66782	3	0.00

The results in Table 3.1 for the generated problems indicate that the first model performs slightly better in terms of expected costs. However the difference in the cost is not significant.

3.6 Conclusions

In this chapter, a new inspection plan has been proposed for the inspection of critical components with several characteristics. A model is formulated that represents the plan. A computational procedure is outlined to obtain the optimal number of repeat inspections that minimizes the total cost. The new model is illustrated with the help of a numerical example. A study is carried out to investigate the effect of sequencing the characteristics on the performance of the model. The results indicate that the model performs slightly better in terms of cost in the case of using the optimal sequence for each cycle of inspection, however the difference is not significant. For implementation, it may be easier to use the model with a fixed order of characteristics at each cycle of inspection.

CHAPTER 4

MATHEMATICAL MODEL FOR THE DEPENDENT CASE

4.1 Introduction

The purpose of this chapter is to extend the model in chapter 3 to the situation where the characteristics' defective rates are statistically dependent. The characteristics' defective rate is not always independent. For example the hardness, strength and toughness of a tool tip are dependent on each other

The same inspection plan as in chapter 3 is used here but the characteristics' defective rates are statistically dependent. To extend the model to this situation requires the knowledge about the joint probability mass function (j.p.m.f.) of the random variables representing characteristics' defective rates. Using j.p.m.f. we can obtain the individual marginal probability mass function. Since the joint and marginal mass functions vary from cycle to cycle, the values for the individual random variable marginal mass function will be updated using Bayes' theorem. After the inspection of the first characteristic, the marginal of other characteristics must be updated prior to inspecting them. Using these concepts, the model 1 in chapter 3 will be modified to handle the case of dependency.

This chapter presents the model for the modified situation and the algorithm proposed for obtaining the number of repeat inspections for the dependent case. The model is described in the next section. Section 3 contains the detailed development of the model. Section 4 and 5 present the algorithm to obtain optimal number of repeat inspections and an example that demonstrates the results of the modified model, respectively. Section 6 describes the results of the two models given in chapter 3 and 4 for the cases where the characteristics' defective rates are independent and dependent. The last section concludes the chapter.

4.2 Model Description

The inspection model is developed for components with several dependent characteristics. A component is rejected if one its characteristic is found to be defective. A component is accepted if all of its characteristics meet the quality specifications. We denote a random variable X_i for the characteristic i, which takes the value 0 if the component is scrap, 1 if the component is to be reworked and 2 if it is good. The joint probability density function of the multivariate random variable $X = (x_1, x_2, \ldots\ldots, x_N)$ is assumed to be known. The probabilities of the misclassifications by the inspector are assumed to be known. Three types of costs are considered: (i) cost of false rejection of an acceptable component (sent to rework or to scrap), (ii) cost of false acceptance of the components which are to be reworked or to be scrapped, and finally, (iii) cost of inspection. The cost of inspection is taken to be of two types i.e. for the inspection at the inspection station and for the inspection at the rework station. The estimates for the three costs are assumed to be available in the industry.

Notations

M_j = Number of components entering the j^{th} cycle of inspection.

N = Number of characteristics in each component to be inspected.

X_i = A discrete random variable which takes value 0 if characteristic i is scrap, 1 if it is rework and 2 if it is nondefective.

$P(x_1, x_2, \ldots, x_N)$ = Joint probability mass function of the random variables X_i, $i = 1,\ldots,N$ at the start of inspection.

$P_i(x_i)$ = Marginal probability mass function of the random variable X_i.

${}^{j}P(x_1, x_2, \ldots, x_N)$ = Joint probability mass function of the random variables X_i for a component entering the j^{th} cycle of inspection.

${}^{kj}P_i(x_i)$ = Marginal probability mass function of the random variable X_i for a component in j^{th} cycle entering the k^{th} stage of inspection.

E_{igr} = Probability of classifying the ith characteristic in the sequence of inspection as rework when it is nondefective.

E_{igs} = Probability of classifying the ith characteristic in the sequence of inspection as scrap when it is nondefective.

E_{irg} = Probability of classifying the ith characteristic in the sequence of inspection as nondefective when it is rework.

E_{irs} = Probability of classifying the ith characteristic in the sequence of inspection as scrap when it is rework.

E_{isg} = Probability of classifying the ith characteristic in the sequence of inspection as nondefective when it is scrap.

E_{isr} = Probability of classifying the ith characteristic in the sequence of inspection as rework when it is scrap.

$M_{i,j}$ = Number of components entering the i^{th} stage of inspection in the j^{th} inspection cycle.

$PG_{i,j}$ = Probability of a component being nondefective entering the i^{th} stage of the j^{th} cycle.

$PR_{i,j}$ = Probability that a component requires rework while entering the i^{th} stage of the j^{th} cycle.

$PS_{i,j}$ = Probability that a component is scrap while entering the i^{th} stage of the j^{th} cycle.

$FR_{i,j}$ = Number of falsely rejected components entering the i^{th} stage of the j^{th} inspection cycle.

$FA_{I,j}$ = Number of falsely accepted components entering the i^{th} stage of the j^{th} inspection cycle.

$CA_{i,j}$ = Number of correctly accepted components entering the i^{th} stage of the j^{th} inspection cycle.

$FGR_{i,j}$ = Number of nondefective components falsely sent to rework in the i^{th} stage of the j^{th} cycle.

$R_{i,j,k}$ = Rate of rejection of components due to i^{th} characteristic at the k^{th} stage of the j^{th} cycle.

A(j) = Number of accepted components in the j^{th} cycle.

CFR(j) = Cost of false rejection in the j^{th} cycle.

CFA(j) = Cost of false acceptance in the j^{th} cycle.

CI(j) = Cost of inspection in the j^{th} cycle.

TCFR	=	Total cost false rejection.
TCFA	=	Total cost false acceptance.
TCI	=	Total cost inspection.
TA	=	Total number of accepted components.
$E(tc)\|_j$	=	Expected total cost per accepted component after j cycles of inspection.
E( )	=	Expected value of the argument inside the parenthesis

4.3 Model Development

We know the joint probability mass functions at the start of inspection for $X_i = 1, 2, \ldots, N$. The individual marginal probability mass functions are given by:

$$P_i(x_i) = \sum_{x_1}\sum_{x_1}\cdots\sum_{x_{i-1}}\sum_{x_{i+1}}\cdots\sum_{x_N} \mathrm{P}(x_1, x_2, \ldots, x_N) \tag{4.1}$$

The marginal probabilities for scrap, rework and good components are computed from equation (4.1) at the start of the inspection to be ${}^1P_i(0)$, ${}^1P_i(1)$ and ${}^1P_i(2)$ respectively, where

$${}^1P_i(2) = 1 - {}^1P_i(0) - {}^1P_i(1) \tag{4.2}$$

After the inspection of first characteristic, the marginal probabilities are updated using Bayes'theorem

$${}^2P_i(0) = \frac{P_i(0)E_{isg}}{[(1-P_i(0)-P_i(1))(1-E_{igs}) + P_i(0)Eisg + P_i(1)E_{irg}]} \tag{4.3}$$

$${}^2P_i(1) = \frac{P_i(1)E_{irg}}{[(1-P_i(0)-P_i(1))(1-E_{igs}) + P_i(0)Eisg + P_i(1)E_{irg}]} \tag{4.4}$$

and

$$^{2}P_i(2) = 1 - {}^{2}P_i(0) - {}^{2}P_i(1) \tag{4.5}$$

In general, we can write for the ith characteristic after being inspected in the jth cycle of inspection:

$$^{j}P_i(0) = \frac{^{j-1}P_i(0)E_{isg}}{[(1 - {}^{j-1}P_i(0) - {}^{j-1}P_i(1))(1 - E_{igs}) + {}^{j-1}P_i(0)E_{isg} + {}^{j-1}P_i(1)E_{irg}]} \tag{4.6}$$

$$^{j}P_i(1) = \frac{^{j-1}P_i(1)E_{irg}}{[(1 - {}^{j-1}P_i(0) - {}^{j-1}P_i(1))(1 - E_{igs}) + {}^{j-1}P_i(0)E_{isg} + {}^{j-1}P_i(1)E_{irg}]} \tag{4.7}$$

and

$$^{j}P_i(2) = 1 - {}^{j}P_i(0) - {}^{j}P_i(1) \tag{4.8}$$

Due to the dependency between characteristics, the marginals of the other characteristics must be updated before inspecting them. For this purpose, we update the joint probabilities first. Therefore, after inspecting ith characteristic in the first cycle the joint probability mass functions are updated as:

$$^{1,1}P(x_1, x_2, \ldots, x_N) = P(x_1, x_2, \ldots, x_N)$$

$$^{2,1}P(x_1, x_2, \ldots, x_N) = {}^{1,1}P(x_1, x_2, \ldots, x_N)\frac{^{2}P_i(x_i)}{^{1}P_i(x_i)} \tag{4.9}$$

i.e. multiply the old joint probability by the ration of the updated and old marginals for the ith characteristic (which is just inspected). From these updated joints we obtain the marginals for the remaining characteristics. Then we go on to inspect the next characteristic in the 2nd stage.

This process is repeated until all the characteristics are inspected. The probability that the component is good, at the end of the cycle is

$$PG_{N,1} = {}^{N,1}P(2,2,.....,2) \tag{4.10}$$

Before any inspection is performed, the total cost of false acceptance is

$$E(tc)|_{j=0} = C_a(1\text{-}P(2,2,.....,2)) \tag{4.11}$$

The expected total cost per accepted component after n cycles of inspection is given by

$$E(tc)|_{j=0} = [TCFR + TCFA + TCI]/TA \tag{4.12}$$

Our objective is to determine the optimal n which minimizes the expected total cost per accepted component.

4.3.1 Analysis of the jth cycle

If $M_{1,j}$ is the number of components entering the first stage of the j[th] cycle,

$$M_{1,j} = M_j$$

The probabilities of a component being nondefective, to be rework or to be scrap are given by

$$PG_{1,j} = P(2,2,.....,2) \tag{4.13}$$

$$PR_{1,j} = \begin{array}{l} \sum\limits_{x_2}\sum\limits_{x_3}....\sum\limits_{x_N} P(1,x_2,x_3,....x_N) + \sum\limits_{x_1}\sum\limits_{x_3}....\sum\limits_{x_N} P(x_1,1,x_3,....x_N) + ...+ \\ \sum\limits_{x_1}\sum\limits_{x_2}....\sum\limits_{x_N} P(x_1,x_2,....x_{N-1},1) - \sum\limits_{x_3}\sum\limits_{x_4}....\sum\limits_{x_N} P(1,1,x_3,....x_N) - .....- \\ \sum\limits_{x_1}\sum\limits_{x_2}....\sum\limits_{x_{N-2}} P(x_1,x_2,....x_{N-2},1,1) + ... \pm P(1,1,1,.....1) \end{array} \tag{4.14}$$

$$PS_{1,j} = 1 - PG_{1,j} - PR_{1,j} \tag{4.15}$$

The probability of the first characteristic to be rework or scrap would be given by

$$P_{1,j}(1) = \sum_{x_2}\sum_{x_3}\sum_{x_3}.....\sum_{x_N} P(1,x_2,x_3,.......,x_N)$$
$$P_{1,j}(0) = \sum_{x_2}\sum_{x_3}\sum_{x_3}.....\sum_{x_N} P(0,x_2,x_3,.......,x_N)$$

Thus the probability of this characteristic to be nondefective would be

$$P_{1j}(2) = 1 - P_{1j}(1) - P_{1j}(0)$$

The components falsely sent to rework are:

$$FR_{1,j} = M_{1,j}(PG_{1,j}E_{1gr} + PS_{1,j}E_{1sr}) \tag{4.16}$$

The components falsely sent to scrap are:

$$FS_{1,j} = M_{1,j}(PG_{1,j}E_{1gs} + PR_{1,j}E_{1rs}) \tag{4.17}$$

The components falsely accepted are:

$$FA_{1,j} = M_{1,j}(P_{1,j}(1)E_{1rg} + P_{1,j}(0)E_{1sg} + (1\text{-}PG_{1,j} - P_{1,j}(1) - P_{1,j}(0)\,)(1\text{-}E_{1gr} - E_{1gs}) \tag{4.18}$$

The components correctly accepted are:

$$CA_{1,j} = M_{1,j}PG_{1,j}\,(1 - E_{1gs} - E_{1gs}) \tag{4.19}$$

All accepted components in this stage move on to the 2nd stage where the next characteristic is inspected. So,

$$M_{2,j} = FA_{1,j} + CA_{1,j} + FGR_{1,j} \tag{4.20}$$

The probabilities that the components bear are

$$PG_{2,j} = {}^{2,j}P(2,2,\ldots..,2) \tag{4.21}$$

$$PR_{2,j} = \begin{array}{l} \sum\limits_{x_2}\sum\limits_{x_3}\cdots\sum\limits_{x_N}{}^{2,j}P(1,x_2,x_3,\ldots x_N)+\sum\limits_{x_1}\sum\limits_{x_3}\cdots\sum\limits_{x_N}{}^{2,j}P(x_1,1,x_3,\ldots x_N)+\ldots..+ \\ \sum\limits_{x_1}\sum\limits_{x_2}\cdots\sum\limits_{x_N}{}^{2,j}P(x_1,x_2,\ldots x_{N-1},1)-\sum\limits_{x_3}\sum\limits_{x_4}\cdots\sum\limits_{x_N}{}^{2,j}P(1,1,x_3,\ldots x_N)-\ldots..\mp \\ \sum\limits_{x_1}\sum\limits_{x_2}\cdots\sum\limits_{x_{N-2}}{}^{2,j}P(x_1,x_2,\ldots x_{N-2},1,1)+\ldots..\pm{}^{2,j}P(1,1,1,\ldots..1) \end{array} \tag{4.22}$$

$$PS_{2,j} = 1 - PG_{2,j} - PR_{2,j} \tag{4.23}$$

The probability of the second characteristic to be rework or scrap would be

$$P_{2,j}(1) = \sum_{x_1}\sum_{x_3}\sum_{x_3}\ldots..\sum_{x_N}{}^{2,j}P(x_1,1,x_3,\ldots\ldots,x_N)$$

$$P_{2,j}(0) = \sum_{x_1}\sum_{x_3}\sum_{x_3}\ldots..\sum_{x_N}{}^{2,j}P(x_1,0,x_3,\ldots\ldots,x_N)$$

and the components are classified as

$$FR_{2,j} = M_{2,j}(PG_{2,j}E_{2gr} + PS_{2,j}E_{2sr}) \tag{4.24}$$

$$FS_{2,j} = M_{2,j}(PG_{2,j}E_{2gs} + PR_{2,j}E_{2rs}) \tag{4.25}$$

$$FA_{2,j} = M_{2,j}[P_{2,j}(1)E_{2rg} + P_{2,j}(0)E_{2sg} + \{1\text{-}PG_{2,j} - P_{2,j}(1) - P_{2,j}(0) + P_{1,j}(1)\ P_{2,j}(1) + P_{1,j}(0)P_{2,j}(0)\}(1\text{-}E_{2gr} - E_{2gs})] \tag{4.26}$$

$$CA_{2,j} = M_{2,j}PG_{2,j}\,(1 - E_{2gs} - E_{2gs}) \tag{4.27}$$

Similarly, we can write for the i^{th} stage of the j^{th} cycle,

$$M_{i,j} = FA_{i\text{-}1,j} + CA_{i\text{-}1,j} + FGR_{I\text{-}1,j} \tag{4.28}$$

$$PG_{i,j} = {}^{i,j}P(2,2,\ldots\ldots,2) \tag{4.29}$$

$$PR_{i,j} = \begin{array}{l} \sum_{x_2}\sum_{x_3}\ldots\sum_{x_N} {}^{i,j}P(1,x_2,x_3,\ldots x_N) + \sum_{x_1}\sum_{x_3}\ldots\sum_{x_N} {}^{i,j}P(x_1,1,x_3,\ldots x_N) + \ldots\ldots + \\ \sum_{x_1}\sum_{x_2}\ldots\sum_{x_N} {}^{i,j}P(x_1,x_2,\ldots x_{N-1},1) - \sum_{x_3}\sum_{x_4}\ldots\sum_{x_N} {}^{i,j}P(1,1,x_3,\ldots x_N) - \ldots\ldots \mp \\ \sum_{x_1}\sum_{x_2}\ldots\sum_{x_{N-2}} {}^{i,j}P(x_1,x_2,\ldots x_{N-2},1,1) + \ldots\ldots \pm {}^{i,j}P(1,1,1,\ldots\ldots 1) \end{array} \tag{4.30}$$

$$PS_{i,j} = 1 - PG_{i,j} - PR_{i,j} \tag{4.31}$$

The probability of the i^{th} characteristic to be rework or scrap would be

$$P_{i,j}(1) = \sum_{x_2}\sum_{x_3}\sum_{x_3}\ldots\ldots\sum_{x_N} {}^{i,j}P(1,x_2,x_3,\ldots\ldots,x_N)$$

$$P_{i,j}(0) = \sum_{x_2}\sum_{x_3}\sum_{x_3}\ldots\ldots\sum_{x_N} {}^{i,j}P(0,x_2,x_3,\ldots\ldots,x_N)$$

$$FR_{i,j} = M_{i,j}(PG_{i,j}E_{igr} + PS_{i,j}E_{isr}) \tag{4.32}$$

$$FS_{i,j} = M_{i,j}(PG_{i,j}E_{Ngs} + PR_{i,j}E_{Nrs}) \tag{4.33}$$

$$FA_{i,j} = M_{i,j}[P_{i,j}(1)E_{irg} + P_{i,j}(0)E_{isg} + \{1\text{-}PG_{i,j} - P_{i,j}(1) - P_{i,j}(0) + \sum_{k=1}^{i-1} P_{k,j}(1)\ P_{i,j}(1) + \sum_{k=1}^{i-2}\sum_{l=k+1}^{i-1} P_{k,j}(1)P_{l,j}(1)P_{i,j}(1) + \ldots\ldots \pm P_{1,j}(1)P_{2,j}(1)\ldots\ldots P_{i,j}(1) + \sum_{k=1}^{i-1} P_{k,j}(0)P_{i,j}(0) +$$

$$\sum_{k=1}^{i-2}\sum_{l=k+1}^{i-1} P_{k,j}(0)P_{l,j}(0)P_{i,j}(0)+\ldots\ldots\pm P_{1,j}(0)P_{2,j}(0)\ldots\ldots P_{i,j}(0)\}(1-E_{igr}-E_{igs})] \quad (4.34)$$

$$CA_{i,j} = M_{i,j}PG_{i,j}\,(1 - E_{igs} - E_{igs}) \quad (4.35)$$

So, in a similar fashion, the next cycle can be analyzed using the updated versions of the joint and marginal probability mass functions. Next, we summarize the results of the j^{th} cycle.

Results of the j^{th} cycle

Total number of accepted components in the j^{th} cycle

$$A(j) = M_{j+1} = CA_{N,j} + FA_{N,j} + FGR_{N,j} \quad (4.36)$$

Total Cost of false acceptance for the jth cycle

$$CFA(j) = C_a FA_{N,j} \quad (4.37)$$

Total Cost of false rejection for the jth cycle

$$CFR(j) = \sum_{i=1}^{N} M_{i,j}(C_{fgs}PG_{i,j}E_{igs} + C_{frs}PR_{i,j}E_{irs}) \quad (4.38)$$

where

C_{fgs} = Cost of misclassifying a nondefective component to be scrap.

C_{frs} = Cost of misclassifying a rework component to be scrap.

Cost of inspection at the inspection station, for the j^{th} cycle

$$CI_1(j) = \sum_{i=1}^{N} C_{i1}M_{i,j} \quad (4.39)$$

Cost of inspection at the rework station, for the j^{th} cycle

$$CI_2(j) = \sum_{i=1}^{N} C_{i2} M_{i,j}\ (PG_{i,j}E_{igr} + PS_{i,j}E_{isr} + PR_{i,j}(1 - E_{irg} - E_{irs})) \tag{4.40}$$

So, the total cost of inspection for the j^{th} cycle is

$$CI(j) = CI_1(j) + CI_2(j) \tag{4.41}$$

Minimization of inspection cost for j^{th} cycle

The total cost of inspection in a cycle is influenced by the sequence in which the characteristics are inspected. The characteristic with lower cost of inspection and higher rate of rejection should be inspected first, as proposed by Duffuaa and Raouf [12] for independent characteristics. In this model, we use the rule given by Duffuaa and Nadeem [13] for the dependent characteristics. So, at the beginning of the j^{th} cycle, we compute the ratio

$$r_{i,j} = C_i/R_{i,j} \tag{4.42}$$

where C_i, the cost of inspection for characteristic i, is given by

$$Ci = C_{i1} + {}^{N,j-1}P_i(1)C_{i2}$$

and the rejection rate for the i^{th} characteristic in the j^{th} cycle is

$$R_{i,j} = {}^{N,j-1}P_i(0)(1 - E_{isg} - E_{isr}) + {}^{N,j-1}P_i(1)E_{irs} + {}^{N,j-1}P_i(2)E_{igs}$$

So, we select the characteristic with the lowest ratio (4.42).

Similarly, at the k^{th} stage of the j^{th} cycle we compute

$$r_{i,j,k} = C_i/R_{i,j,k} \tag{4.43}$$

where

$$Ci = Ci1 + {}^{k-1,j}P_i(1)C_{i2}$$

and

$$R_{i,j,k} = {}^{k-1,j}P_i(0)(1 - E_{isg} - E_{isr}) + {}^{k-1,j}P_i(1)E_{irs} + {}^{k-1,j}P_i(2)E_{igs}$$

and select the one among the remaining characteristics with the lowest ratio (4.42) to be inspected at the k^{th} stage.

Total cost after n cycles

After analyzing n cycles, we compute total cost of false acceptance TCFA, total cost of false rejection TCFR, total cost of inspection TCI and the total number of accepted components TA, as

$$TCFA = CFA(n) = C_a FA_{N,n} \tag{4.44}$$

$$TCFR = \sum_{j=1}^{n} [CFR(j)] \tag{4.45}$$

$$TCI = \sum_{j=1}^{n} [CI(j)] \tag{4.46}$$

$$TA = CA_{N,n} + FA_{N,n} \tag{4.47}$$

Substituting TCFA, TCFR, TCI and TA in (12) we obtain the total expected cost at the end of n^{th} cycle. Next is given a computational procedure to determine optimal n.

4.4 Computational Procedure

Step 1. Compute $E(tc)|j=0$ using (4.11). Set j=1.

Step 2. Compute ${}^{j}P_i(0)$, ${}^{j}P_i(1)$, ${}^{j}P_i(2)$ and M_j using (4.6), (4.7), (4.8) and (4.36) respectively. Select the i^{th} characteristic for inspection based on the ratio (4.43). Repeat this until all the characteristics are inspected.

Step 3. Compute Mi,j, FSi,j, FAi,j and CAi,j using (4.28), (4.33), (4.34), (4.35) respectively.

Step 4. Compute CFA(j), CFR(j) and CI(j) using (4.37), (4.38) and (4.41) respectively.

Step 5. Compute TCFA(j), TCFR(j), TCI(j) and TA(j) using (4.44), (4.45), (4.46) and (4.47) respectively.

Step 6. Compute E(tc)|j using (4.12).

Step 7. If E(tc)|j < E(tc)|j-1, set j=j+1 and go to step 2. Otherwise n = j-1.

4.5 Illustrative Example

In order to illustrate the model presented in this chapter, the following example is provided. A program is developed implementing the algorithm in section (4.4) and is given in Appendix (B). It is used to obtain the optimal number of repeat inspections.

Example:

Assuming that the following data is given

		I								
		0			1			2		
		II			**II**			**II**		
		0	1	2	0	1	2	0	1	2
III	0	0.004	0.006	0.00025	0.002	0.004	0.108	0.004	0.072	0.00075
	1	0.006	0.0045	0.032	0.002	0.004	0.032	0.00025	0.00025	0.00025
	2	0.0135	0.072	0.0135	0.00075	0.036	0.00075	0.0045	0.00075	0.576

$$N = 3,\ M = 100,\ C_a = 100000,\ C_{i1} = 100,\ C_{i2} = 5000,\ C_{fgs} = 10000,\ C_{frs} = 5000$$

$$E_{gs} = 0.03,\ E_{sg} = 0.1,\ E_{gr} = E_{rg} = E_{sr} = E_{rs} = 0.1$$

Solving this example using the proposed model gives the following results:

Expected Total Cost per accepted component without inspection = 42400.00

______________ Cycle 1 ____________________________

PG(1) 5.760000E-01

A(1) 51

Etc(1) 8879.026000

______________ Cycle 2 ____________________________

PG(2) 9.895343E-01

A(2) 45

Etc(2) 11886.050000

Optimal no. of Inspection = 1

Thus the optimal number of inspection cycles is 1 when the characteristics' defective rates are statistically dependent with the given data. It can be noticed that the defective rates for the illustrative example in chapter 3 are independent while the ones in the above example are dependent. Next we compare the results of the two models for the two cases i.e. the dependent and the independent one.

4.6 Comparison of the Models

If X and Y are two random variables, with joint probability distribution f(x,y) and marginal distribution g(x) and h(y), respectively, then the random variable X and Y are said to be statistically independent if

$$f(x,y) = g(x)h(y) \tag{4.48}$$

Thus we choose the defective rates for the two models that follow above condition and examine their effect on the results.

Assuming that the following data is given for the two models described in chapter 3 and chapter 4 respectively:

			I								
			0			1			2		
			0.1			0.1			0.8		
			II			**II**			**II**		
			0	1	2	0	1	2	0	1	2
			0.05	0.05	0.9	0.05	0.05	0.9	0.05	0.05	0.9
III	0	0.15	0.00075	0.00075	0.0135	0.00075	0.00075	0.0135	0.006	0.006	0.108
	1	0.05	0.00025	0.00025	0.0045	0.00025	0.00025	0.0045	0.002	0.002	0.036
	2	0.8	0.004	0.004	0.072	0.004	0.004	0.072	0.032	0.032	0.576

$$N = 3,\ M = 100,\ C_a = 100000,\ C_{i1} = 100,\ C_{i2} = 5000,\ C_{fgs} = 10000,\ C_{frs} = 5000$$

$$E_{gs} = 0.03,\ E_{sg} = 0.1,\ E_{gr} = E_{rg} = E_{sr} = E_{rs} = 0.05$$

Solving this example using the proposed models gives the following results:

Model 1:

Expected Total Cost per accepted component without inspection = 42400.00

______________ Cycle 1 ____________________________

PG(1) 0.5760000

A(1) 53

Etc(1) 10798.160000

______________ Cycle 2 ____________________________

PG(2) 0.9512286

A(2) 45

Etc(2) 9439.654000

______________ Cycle 3 ____________________________

PG(3) 0.9955484

A(3) 39

Etc(3) 12580.210000

Optimal no. of Inspections = 2

Model 2

Expected Total Cost per accepted component without inspection = 42400.00

______________ Cycle 1 ______________

PG(1) 0.5760000

A(1) 53

Etc(1) 10798.160000

______________ Cycle 2 ______________

PG(2) 0.9512286

A(2) 45

Etc(2) 9439.654000

______________ Cycle 3 ______________

PG(3) 0.9954568

A(3) 39

Etc(3) 12581.830000

Optimal no. of Inspection = 2

It can be observed that the two models perform identically under the independence condition, in terms of optimal expected total cost, number of cycles of inspection and outgoing quality.

Next we examine the results of the two models under the dependence condition that is the characteristics' defective rates don't follow the condition given in (4.48).

Assuming that the following data is given for the two models described in chapter 3 and chapter 4 respectively:

			I								
			0			1			2		
			0.15175			0.18950			0.65875		
			II			II			II		
			0	1	2	0	1	2	0	1	2
			0.0370	0.1995	0.7635	0.0370	0.1995	0.7635	0.0370	0.1995	0.7635
III	0	0.20100	0.004	0.006	0.00025	0.002	0.004	0.108	0.004	0.072	0.00075
	1	0.08125	0.006	0.0045	0.032	0.002	0.004	0.032	0.00025	0.00025	0.00025
	2	0.71775	0.0135	0.072	0.0135	0.00075	0.036	0.00075	0.0045	0.00075	0.576

$$N = 3,\ M = 100,\ C_a = 100000,\ C_{i1} = 100,\ C_{i2} = 5000,\ C_{fgs} = 10000,\ C_{frs} = 5000$$

$$E_{gs} = 0.03,\ E_{sg} = 0.1,\ E_{gr} = E_{rg} = E_{sr} = E_{rs} = 0.05$$

Solving this example using the proposed models gives the following results:

Model 1:

Expected Total Cost per accepted component without inspection = 63900.36

__________ Cycle 1 __________

PG(1) 0.3639042

A(1) 34

Etc(1) 22022.07

__________ Cycle 2 __________

PG(2) 0.9136896

A(2) 27

Etc(2) 19817.65

_____________ Cycle 3 ___________________________

PG(3) 0.9924110

A(3) 24

Etc(3) 23640.32

Optimal no. of Inspections = 2

Model 2

Expected Total Cost per accepted component without inspection = 42400.00

_____________ Cycle 1 ___________________________

PG(1) 0.5760000

A(1) 52

Etc(1) 7772.64

_____________ Cycle 2 ___________________________

PG(2) 0.9932351

A(2) 46

Etc(2) 10160.76

Optimal no. of Inspection = 1

It can be observed that under the case of dependence of the chracteristics' defective rates, model 2 performs better in all terms i.e. optimal expected total cost, number of cycles of inspection and the average outgoing quality.

The above conclusion is further illustrated with the help of following four more examples. The probabilities of rework and scrap in these problems are varied between 0.05 and 0.2 to obtain different sets of data for the independent problems. To obtain the sets of data for the dependent problems the respective set of the joint probabilities of the independent problem are shuffled. However, the other parameters remain the same as in Example 1.

Example 2:

Independent Problem

			I								
			0			1			2		
			0.15			0.1			0.75		
			II			**II**			**II**		
			0	1	2	0	1	2	0	1	2
			0.1	0.1	0.8	0.1	0.1	0.8	0.1	0.1	0.8
III	0	0.05	0.00075	0.00075	0.006	0.0005	0.0005	0.004	0.00375	0.00375	0.03
	1	0.05	0.00075	0.00075	0.006	0.0005	0.0005	0.004	0.00375	0.00375	0.03
	2	0.9	0.0135	0.0135	0.108	0.009	0.009	0.072	0.0675	0.0675	0.54

Results:

Model 1

Etc(0) 46000.000000

PG(1) 0.5400000
A(1) 51
Etc(1) 11736.060000

PG(2) 0.9466554
A(2) 44
Etc(2) 9851.473000

PG(3) 0.9951380
A(3) 40
Etc(3) 12456.770000

Optimal no. of Inspection = 2

Model 2

Etc(0) 46000.000000

PG(1) 0.5400000
A(1) 51
Etc(1) 11736.050000

PG(2) 0.9466554
A(2) 44
Etc(2) 9851.473000

PG(3) 0.9951380
A(3) 40
Etc(3) 12456.770000

Optimal no. of Inspection = 2

Dependent Problem

			I								
			0			1			2		
			0.1325			0.0525			0.815		
			II			II			II		
			0	1	2	0	1	2	0	1	2
			0.1	0.1	0.8	0.1	0.1	0.8	0.1	0.1	0.8
III	0	0.2325	0.009	0.009	0.0072	0.00075	0.00075	0.006	0.0135	0.0135	0.108
	1	0.08	0.00375	0.00375	0.03	0.00375	0.00375	0.03	0.0005	0.0005	0.004
	2	0.6875	0.0005	0.0005	0.004	0.00075	0.00075	0.006	0.0675	0.0675	0.54

Results:

Model 1

Etc(0) 55175.000000

PG(1) 0.4482500
A(1) 43
Etc(1) 14566.7400

PG(2) 0.9239689
A(2) 37
Etc(2) 10805.040000

PG(3) 0.9925753
A(3) 33
Etc(3) 13283.550000

Optimal no. of Inspection = 2

Model 2

Etc(0) 46000.000000

PG(1) 0.5400000
A(1) 50
Etc(1) 9861.403000

PG(2) 0.9578044
A(2) 44
Etc(2) 8778.723000

PG(3) 0.9962031
A(3) 40
Etc(3) 11371.810000

Optimal no. of Inspection = 2

Example 3:

Independent Problem

			I								
			0			1			2		
			0.1			0.2			0.7		
			II			II			II		
			0	1	2	0	1	2	0	1	2
			0.05	0.05	0.9	0.05	0.05	0.9	0.05	0.05	0.9
III	0	0.1	0.0005	0.0005	0.009	0.001	0.001	0.018	0.0035	0.0035	0.063
	1	0.15	0.00075	0.00075	0.0135	0.0015	0.0015	0.027	0.00525	0.00525	0.0945
	2	0.75	0.00375	0.00375	0.0675	0.0075	0.0075	0.135	0.02625	0.02625	0.4725

Results:

Model 1

Etc(0) 52750.000000

PG(1) 0.4725000
A(1) 44
Etc(1) 16988.120000

PG(2) 0.9404901
A(2) 38
Etc(2) 15323.730000

PG(3) 0.9950531
A(3) 35
Etc(3) 18471.670000

Optimal no. of Inspection = 2

Model 2

Etc(0) 52750.000000

PG(1) 0.4725000
A(1) 44
Etc(1) 16988.120000

PG(2) 0.9404900
A(2) 38
Etc(2) 15323.730000

PG(3) 0.9950530
A(3) 35
Etc(3) 18471.670000

Optimal no. of Inspection = 2

Dependent Problem

			I								
			0			1			2		
			0.24			0.13			0.63		
			II			II			II		
			0	1	2	0	1	2	0	1	2
			0.05	0.05	0.9	0.05	0.05	0.9	0.05	0.05	0.9
III	0	0.2	0.001	0.001	0.018	0.00525	0.00525	0.0945	0.00375	0.00375	0.0675
	1	0.11	0.0035	0.0035	0.063	0.0005	0.0005	0.009	0.0015	0.0015	0.027
	2	0.69	0.0075	0.0075	0.135	0.00075	0.00075	0.0135	0.02625	0.02625	0.4725

Results:

Model 1

Etc(0) 60877.000000

PG(1) 0.3912300
A(1) 37
Etc(1) 16399.530000

PG(2) 0.9096909
A(2) 32
Etc(2) 11960.460000

PG(3) 0.9912164
A(3) 29
Etc(3) 14452.440000

Optimal no. of Inspection = 2

Model 2

Etc(0) 52750.000000

PG(1) 0.4725000
A(1) 45
Etc(1) 11908.320000

PG(2) 0.9435249
A(2) 39
Etc(2) 9950.514000

PG(3) 0.9944924
A(3) 35
Etc(3) 12509.140000

Optimal no. of Inspection = 2

Example 4:

Independent Problem

			I								
			0			1			2		
			0.05			0.15			0.8		
			II			II			II		
			0	1	2	0	1	2	0	1	2
			0.15	0.1	0.75	0.15	0.1	0.75	0.15	0.1	0.75
III	0	0.1	0.00075	0.0005	0.00375	0.00225	0.0015	0.01125	0.012	0.008	0.06
	1	0.2	0.0015	0.001	0.0075	0.0045	0.003	0.0225	0.024	0.016	0.12
	2	0.7	0.00525	0.0035	0.02625	0.01575	0.0105	0.07875	0.084	0.056	0.42

Results:

Model 1

Etc(0) 58000.000000

PG(1) 0.4200000
A(1) 40
Etc(1) 18041.520000

PG(2) 0.9304101
A(2) 34
Etc(2) 15721.230000

PG(3) 0.9941040
A(3) 31
Etc(3) 18840.400000

Optimal no. of Inspection = 2

Model 2

Etc(0) 58000.000000

PG(1) 0.4200000
A(1) 40
Etc(1) 18041.520000

PG(2) 0.9304101
A(2) 34
Etc(2) 15721.230000

PG(3) 0.9941040
A(3) 31
Etc(3) 18840.400000

Optimal no. of Inspection = 2

Dependent Problem

			I								
			0			1			2		
			0.195			0.13			0.675		
			II			II			II		
			0	1	2	0	1	2	0	1	2
			0.15	0.1	0.75	0.15	0.1	0.75	0.15	0.1	0.75
III	0	0.3	0.024	0.016	0.12	0.00525	0.0035	0.02625	0.01575	0.0105	0.07875
	1	0.095	0.00075	0.0005	0.00375	0.012	0.008	0.06	0.0015	0.001	0.0075
	2	0.605	0.0045	0.003	0.0225	0.00225	0.0015	0.01125	0.084	0.056	0.42

Results:

Model 1

Etc(0) 69371.880000

PG(1) 0.3062813
A(1) 31
Etc(1) 23293.080000

PG(2) 0.8837426
A(2) 25
Etc(2) 16162.680000

PG(3) 0.988344
A(3) 23
Etc(3) 18850.750000

Optimal no. of Inspection = 2

Model 2

Etc(0) 58000.000000

PG(1) 0.4200000
A(1) 50
Etc(1) 31147.880000

PG(2) 0.7343974
A(2) 43
Etc(2) 9041.025000

PG(3) 0.9948242
A(3) 39
Etc(3) 11507.150000

Optimal no. of Inspection = 2

Example 5:

Independent Problem

			I								
			0			1			2		
			0.07			0.1			0.83		
			II			II			II		
			0	1	2	0	1	2	0	1	2
			0.1	0.05	0.85	0.1	0.05	0.85	0.1	0.05	0.85
III	0	0.12	0.00084	0.00042	0.00714	0.0012	0.0006	0.0102	0.00996	0.0050	0.08466
	1	0.13	0.00091	0.000455	0.007735	0.0013	0.00065	0.01105	0.01079	0.0054	0.09172
	2	0.75	0.00525	0.002625	0.044625	0.0075	0.00375	0.06375	0.06225	0.0311	0.52913

Results:

Model 1

Etc(0) 47087.500000

PG(1) 0.5291250
A(1) 50
Etc(1) 12664.400000

PG(2) 0.9465292
A(2) 43
Etc(2) 11059.660000

PG(3) 0.9952313
A(3) 39
Etc(3) 13794.780000

Optimal no. of Inspection = 2

Model 2

Etc(0) 47087.500000

PG(1) 0.5291250
A(1) 50
Etc(1) 12664.400000

PG(2) 0.9465292
A(2) 43
Etc(2) 11059.660000

PG(3) 0.9952313
A(3) 39
Etc(3) 13794.780000

Optimal no. of Inspection = 2

Dependent Problem

			I								
			0			1			2		
			0.1329			0.1605			0.7066		
			II			II			II		
			0	1	2	0	1	2	0	1	2
			0.1	0.05	0.85	0.1	0.05	0.85	0.1	0.05	0.85
III	0	0.1876	0.0013	0.00065	0.01105	0.00996	0.00498	0.08466	0.0075	0.00375	0.06375
	1	0.0295	0.0012	0.0006	0.0102	0.00084	0.00042	0.00714	0.00091	0.00046	0.00774
	2	0.7829	0.01079	0.00540	0.09172	0.00525	0.00263	0.04463	0.06225	0.03113	0.52913

Results:

Model 1

Etc(0) 52978.240000

PG(1) 0.4702176
A(1) 45
Etc(1) 15068.190000

PG(2) 0.9305602
A(2) 38
Etc(2) 11963.950000

PG(3) 0.9933739
A(3) 35
Etc(3) 14627.140000

Optimal no. of Inspection = 2

Model 2

Etc(0) 47087.500000

PG(1) 0.5291250
A(1) 50
Etc(1) 11392.350000

PG(2) 0.9502838
A(2) 43
Etc(2) 9532.832000

PG(3) 0.9952183
A(3) 39
Etc(3) 12106.820000

Optimal no. of Inspection = 2

4.7 Conclusions

In this chapter, a new inspection model has been designed for the inspection of critical components with several characteristics. This model is a generalization of the model given in chapter 3 in that it deals with dependent characteristics' defective rates. A computational procedure is outlined to obtain the optimal number of repeat inspections that minimizes the total cost. The new model is illustrated with the help of a numerical example. The performance of the two models is compared with the help of five examples each for the case where the characteristics' defective rates are statistically dependent and independent. The comparison for the case where characteristics' defective rates are statistically independent shows that the two models perform identically while in case of dependent charateristics' defective rates, model 2 performs well in terms of the optimal expected total cost of inspection, number of cycles of inspection and the average outgoing quality.

CHAPTER 5

ECONOMIC EFFECTS OF INSPECTION ERROR

5.1 Introduction

Inspection is a process of comparing or determining the conformance of products to established specifications. The accuracy of inspection process is the key element for better product quality and the process of inspection by some individual can never be 100% error free. There would always be a substantial fraction of the total cost of inspection associated with the misclassifications made by the inspectors. Therefore, the thirst to identify the most significant error remains persistent whenever a new inspection model is considered.

This chapter presents the study of the effect of the inspector error on the repeat inspection plan for the two models developed in this thesis. The measures of performance for this study were taken to be the expected total cost, average total inspection and the average outgoing quality. The study is carried out for a specific range of inspection error. Section 5.2 presents the results of the study for the model 1 and section 5.3 describes the results of the study for the model 2. Section 5.4 gives the analysis of these results. Section 5.5 concludes the chapter.

5.2 Economic Effects of Inspection Error on Model 1

To inspect the effects of the errors Esg (i.e. misclassifying a scrap component to be good) and Egs (i.e. misclassifying a good component to be scrap) a batch of 100 components, each with 3 characteristics was inspected with incoming quality known i.e. probability of the component to be reworked being 0.10, 0.05, 0.05 and that of scrap being 0.10, 0.05 and 0.15 respectively. The cost of inspection for the three characteristics is taken to be 100 at the inspection station and 5000 at the rework station. The cost of false acceptance is 100000. The cost of misclassification of good component to scrap is 10000 and that of reworkable one to scrap is 5000. The two errors i.e. misclassifying good component to scrap or a scrap one to be good were taken to be 0.01, 0.03, 0.05, 0.10 and 0.15 to observe the effect on expected total cost per accepted component (ETC), average total inspection (ATI) and the average outgoing quality AOQ. This range of errors is obtained from [4,10] and the valued of the costs are obtained from [10]. Fixing the other errors of misclassification at any of the above five levels, the model was run 25 times each for the different values of Esg and Egs. The model determines the optimal number of repeat inspections and simulates the inspection process for each pairs of the errors. The results of the runs for the five levels of the other errors of misclassifications are shown in Tables A.1 to A.5 in Appendix (A). Each of these tables shows the pairs of errors and optimal values for ETC, AOQ which is (1 - PG), n and A(n) for each pair of errors. Also the tables show the total cost of false rejection (TCFR), total cost of false acceptance (TCFA) and total cost of inspection (TCI). All of these costs are taken per accepted component. Figures 5.1- 5.10 show the effect of the variation in Esg and Egs on AOQ and ATI at different levels of the other errors of misclassification. Next, we examine the impact of the errors on these two performance measures of the inspection plan.

5.2.1 Impact of Inspection Errors on Average Outgoing Quality

The average quality of the lot going out of the inspection process is defined as

$$\text{AOQ} = \frac{\text{Number of defective components after inspection}}{\text{Total number of accepted components}}$$

Figure 5.1 contains five curves. Each curve shows the relationship between Esg and AOQ for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.01. It is observed that an AOQ increase as Egs increases, i.e. the expected number of good components decreases. For Egs levels of 0.10-0.15 the increase in AOQ is similar when Esg varies from 0.01 to 0.15. However, at Egs level of 0.03 and 0.05, AOQ decreases by 81% when Esg goes from 0.05 to 0.10. These unusual phenomena can be justifies from Table A.1. It is due to the fact that AOQ gets so higher that the model in ordr to minimize AOQ, is forced to take one more cycle of inspection. It can be viewed that the rate of increase in AOQ goes up with the leel of Egs, shich is in line with the intuition.

Figure 5.2 contains five curves. Each curve shows the relationship between Esg and AOQ for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.03. It is observed that an AOQ increase as Egs increases, which is quite in line with the perception. For Egs levels of 0.10-0.15 the increase in AOQ is similar when Esg varies from 0.01 to 0.15. However, at Egs level of 0.03 and 0.05, AOQ decreases by 84% when

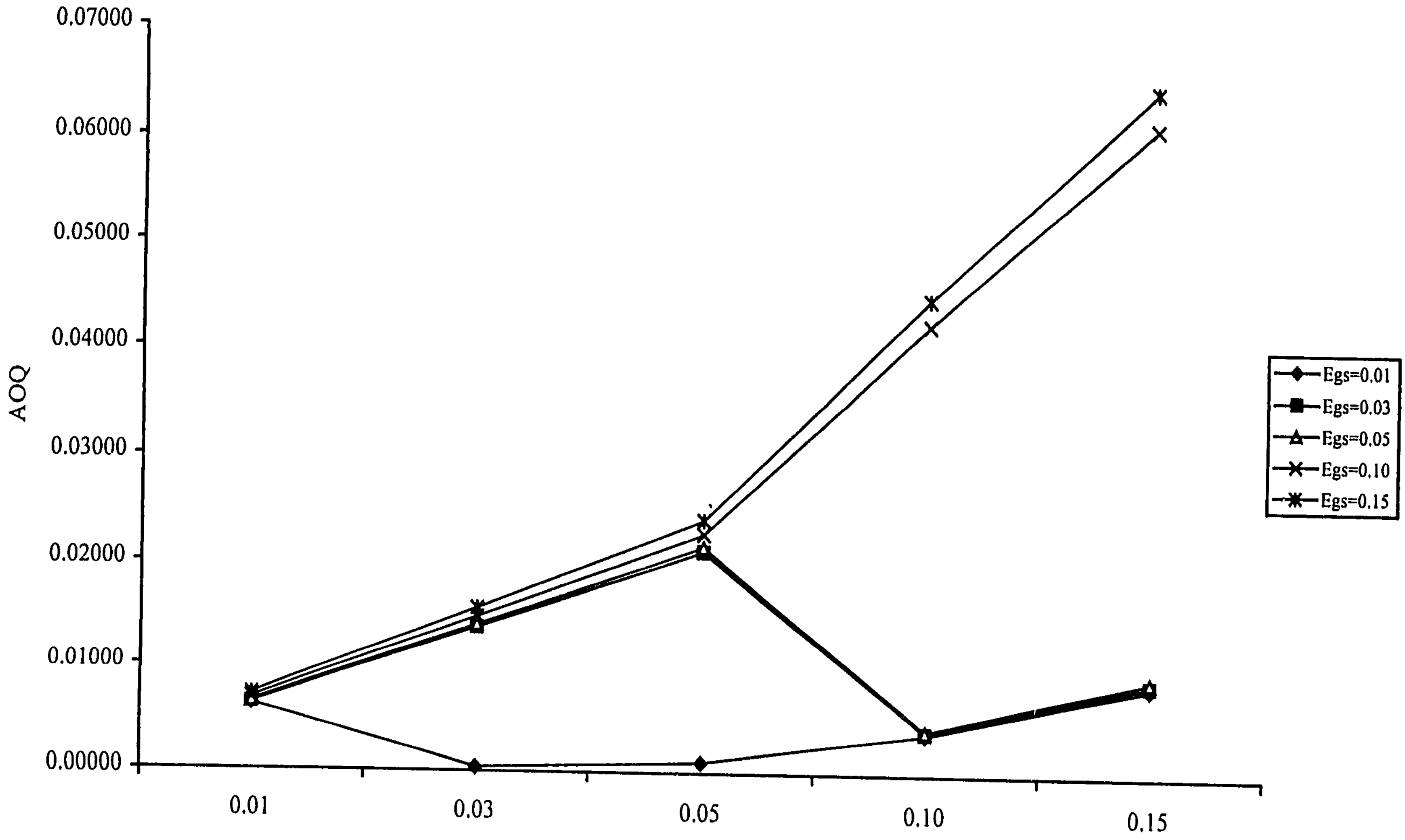

Figure 5.1 Effect of the error on the Average Outgoing Quality (AOQ) in Model 1

Egr = Erg = Ers = Esr = 0.01

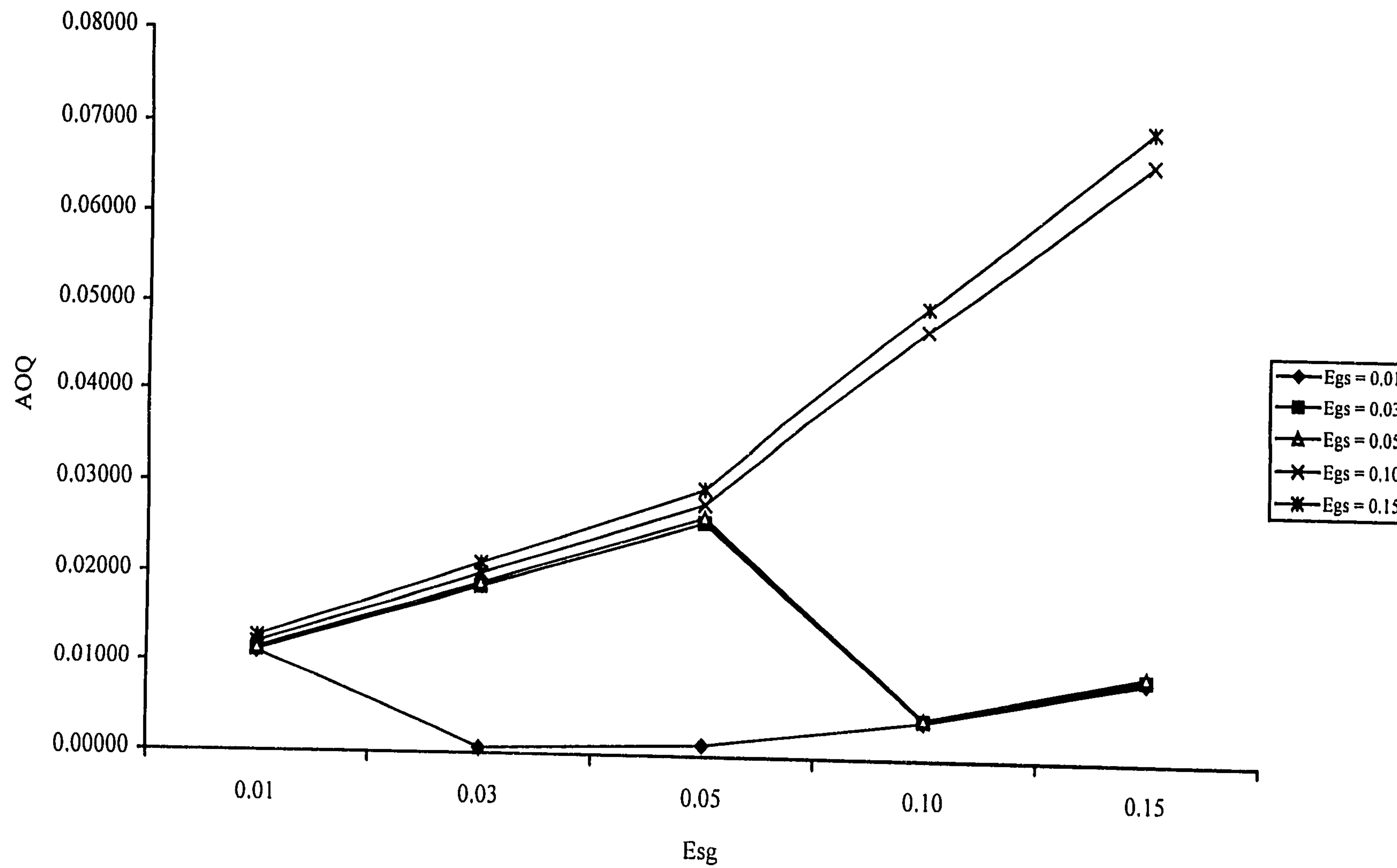

Figure 5.2 Effect of the error Esg on the Average Outgoing Quality (AOQ) in Model 1
Egr = Erg = Ers = Esr = 0.03

Esg goes from 0.05 to 0.10. Theis unusual behavior is due to the reason that AOQ gets very high and the model, to attain lower values of AOQ takes one more cycle of inspection. It can be perceived that the rate of increase in AOQ goes up with the level of Egs, which is in line with the intuition.

Figure 5.3 contains five curves. Each curve shows the relationship between Esg and AOQ for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.05. It is observed that an AOQ increase as Egs increases, which is something one can expect by intuition. The decrease in AOQ at Egs level of 0.05,when Esg goes from 0.10 to 0.15 is 80.7%, and the decrease in AOQ at Egs level of 0.03,when Esg goes from 0.05 to 0.10 is 85.6%. These unusual phenomena can be justifies from Table A.3. It is due to the fact that AOQ gets so higher that the model in ordr to minimize AOQ, is forced to take one more cycle of inspection. The figure shows that the more the level of Egs the greater will be the change in AOQ.

Figure 5.4 contains five curves. Each curve shows the relationship between Esg and AOQ for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.10. It is observed that an AOQ increase as Egs increases, which is quite conceptual. The decrease in AOQ at Egs level of 0.05,when Esg goes from 0.10 to 0.15 is 80.9%, and the decrease in AOQ at Egs level of 0.03,when Esg goes from 0.05 to 0.10 is 84.8%. These unusual phenomena are due to the reason that AOQ gets so higher that the model in order to minimize AOQ, is forced to take one more cycle of inspection. It can be observed that the rate of increase in AOQ goes up with the level of Egs.

Similarly, Figure 5.5 contains five curves. Each curve shows the relationship between Esg and AOQ for a fixed level of Egs. The other errors of misclassifications are taken to

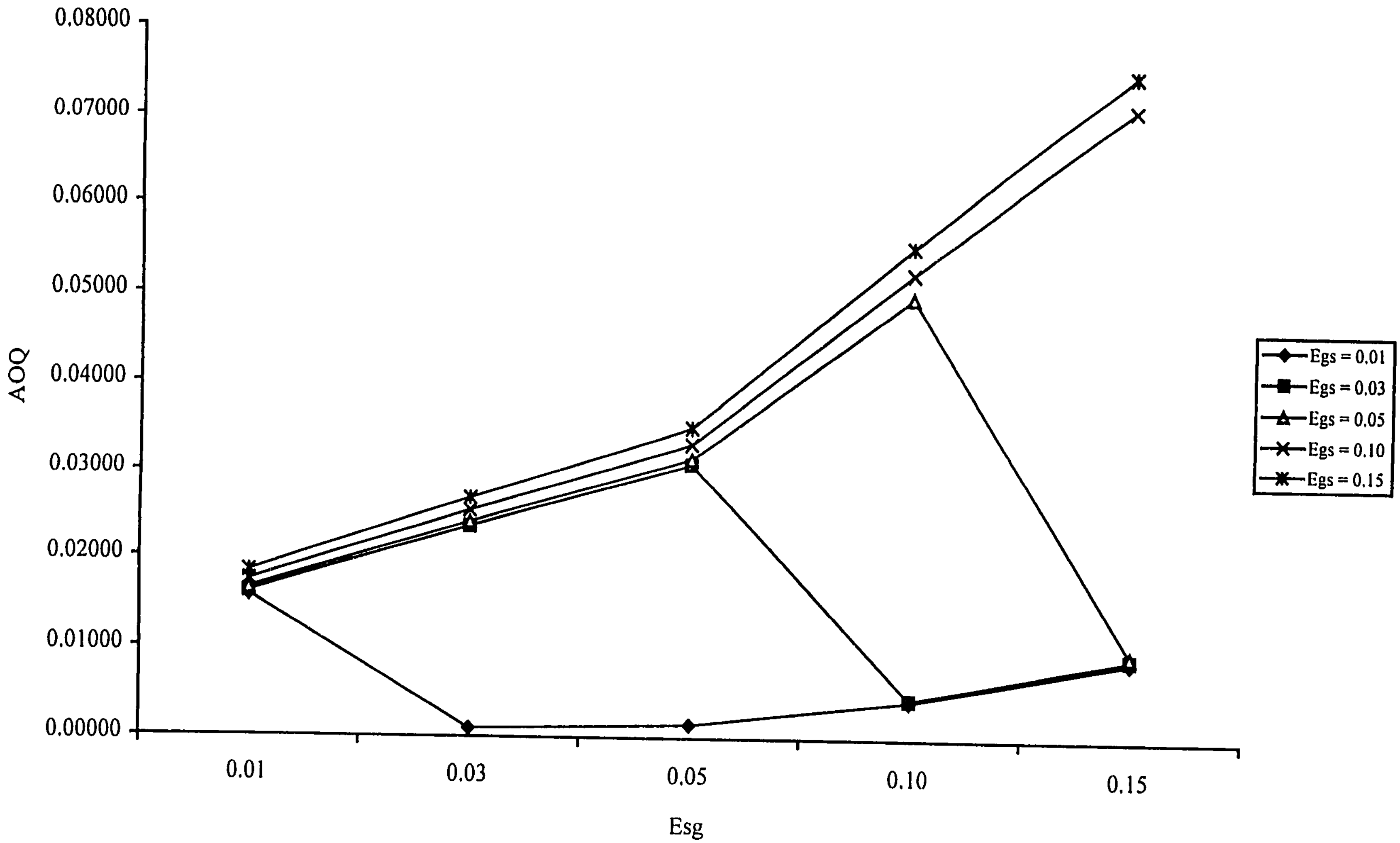

Figure 5.3 Effect of the error Esg on the Average Outgoing Quality (AOQ) in Model 1
Egr = Erg = Ers = Esr = 0.05

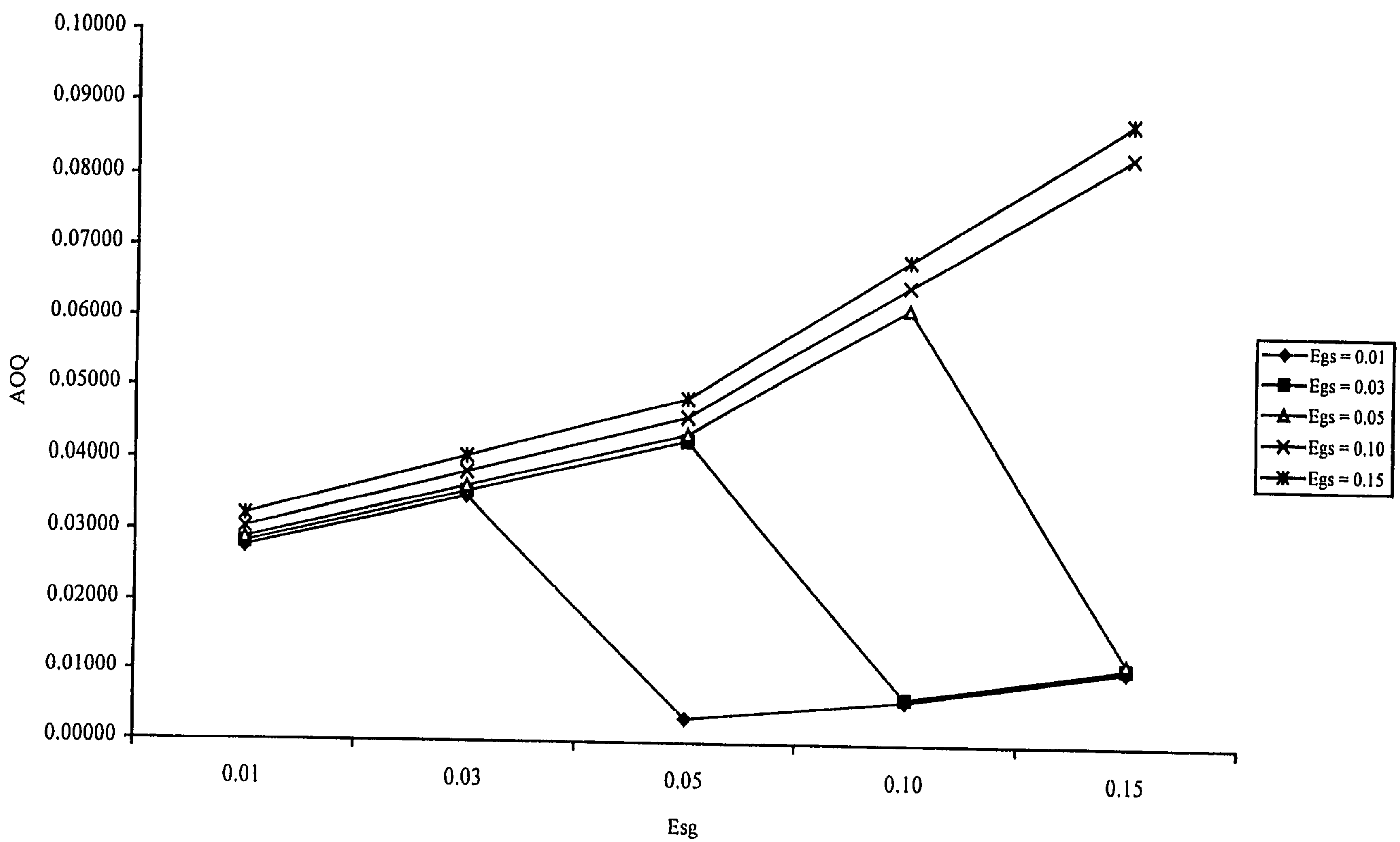

Figure 5.4 Effect of the error Esg on the Average Outgoing Quality (AOQ) in Model 1
Egr = Erg = Ers = Esr = 0.10

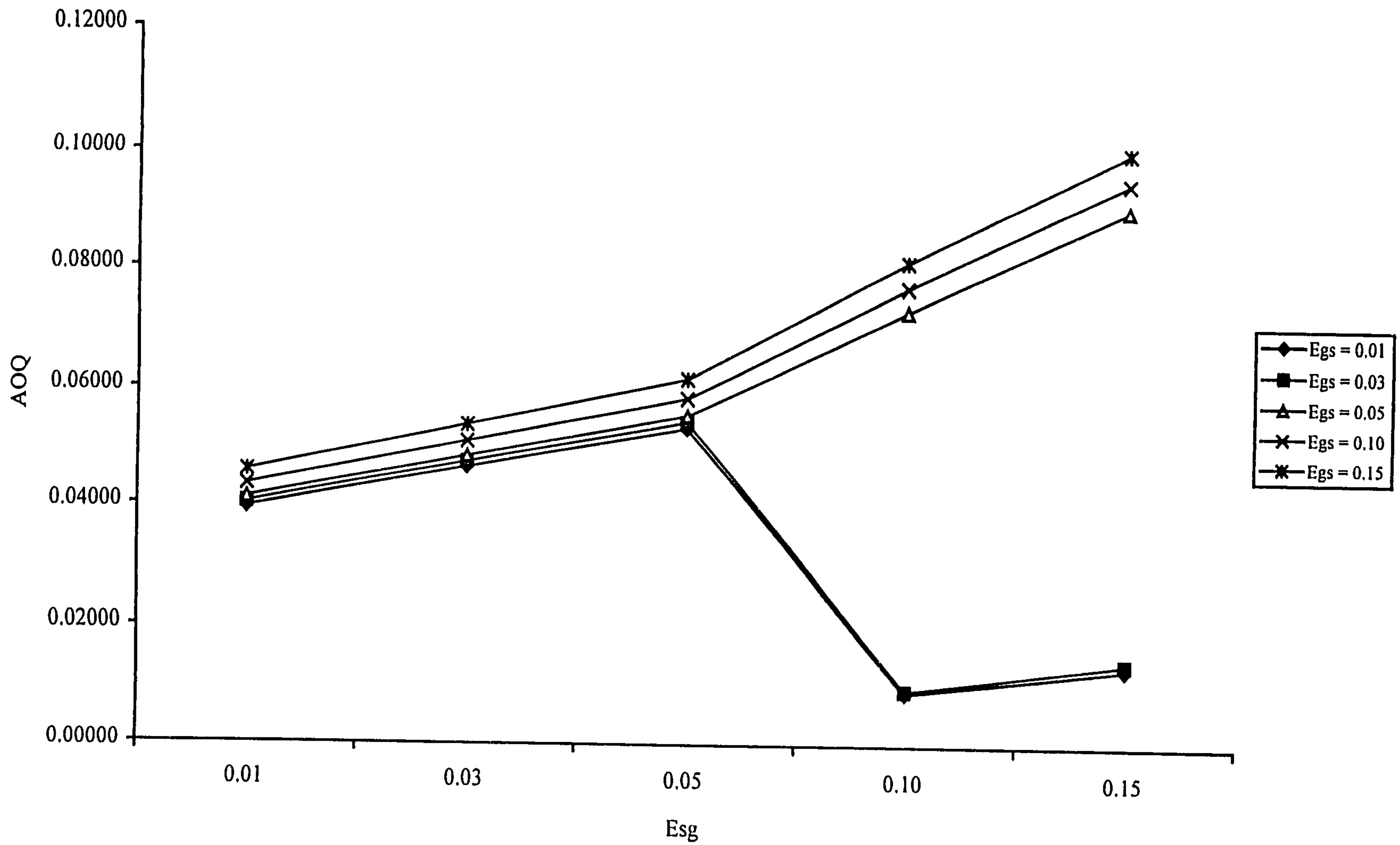

Figure 5.5 Effect of the error Esg on the Average Outgoing Quality (AOQ) in Model 1
Egr = Erg = Ers = Esr =0.15

be at level 0.15. It is observed that an AOQ increase as Egs increases, that is again, the more we misclassify the lesser number of good items we will achieve. However, at Egs level of 0.01 or 0.03, AOQ decreases by 82% when Esg goes from 0.05 to 0.10. This unusual phenomenon can be justified from Table A.5. It is due to the fact that AOQ gets so higher that the model in order to minimize AOQ, is forced to take one more cycle of inspection. The figure shows that the more the level of Egs the greater will be the change in AOQ, which is true by intuition.

In general it can be concluded that AOQ level goes up with the increase in Egs and the amount of this increase in AOQ for increasing Esg, decreases as the level of the other errors of misclassification increase the level of AOQ.

5.2.2 Impact of Inspection Errors on Average Total Inspection

The average total inspection is defined as the total number of inspections conducted in the optimal inspection plan. The inspection plan in the thesis is such that inspection is carried out at the characteristic-inspection-stations and at the rework stations. Thus for a batch of M components ATI is computed as:

$$\mathrm{ATI} = \sum_{j=i}^{n}\left(\sum_{i=1}^{N}\left[\ M_{i,j} + M_j{}^{i}PG\left[\prod_{k=0}^{i-1}(1-E_{kgs})\ \right]E_{igr} + M_{i,j}PS_{i,j}E_{isr} + M_{i,j}PR_{i,j}(1-E_{irg}-E_{irs})\right]\right)$$

ATI can be interpreted as the inspection load in the plan.

Figure 5.6 contains five curves. Each curve shows the relationship between Esg and ATI for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.01. It is observed that the inspection load decreases with the increase in Egs, i.e. the

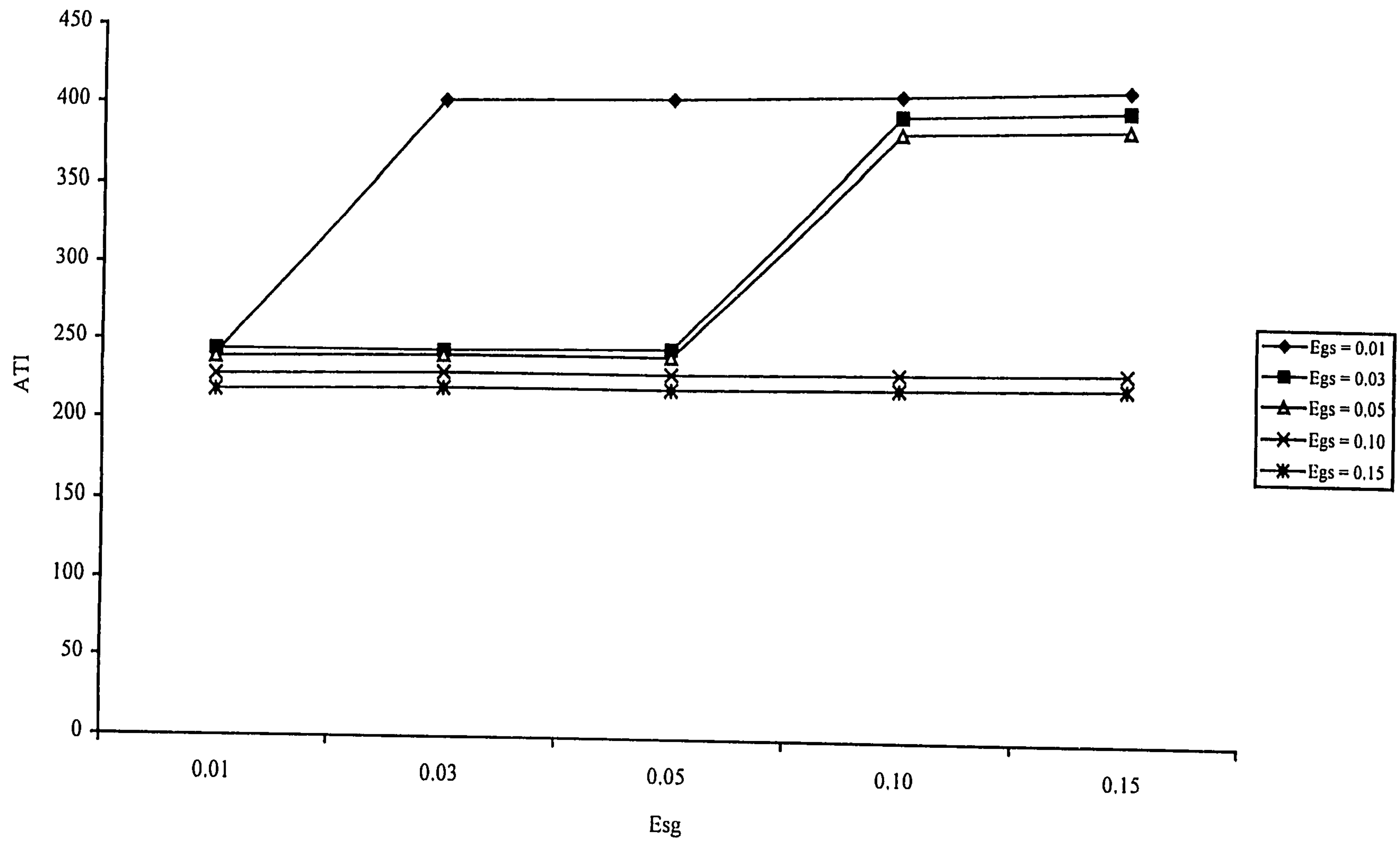

Figure 5.6 Effect of the error on the Average Total Inspection (ATI) in Model 1
Egr = Erg = Ers = Esr =0.01

more we misclassify the good components the lesser we would have to inspect. On the other hand, inspection load increases in a piece wise linear fashion as Esg increases. At Egs level of 0.01, ATI increases by 67.2% by altering Esg from 0.01 to 0.03. However, ATI increases by 61% at Egs level of 0.03 for Esg varying from 0.05 to 0.10. The reason for these unusual phenomena is that higher level of AOQ forces the model to take one more cycle of inspection.

Figure 5.7 contains five curves. Each curve shows the relationship between Esg and ATI for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.03. It is observed that the inspection load decreases with the increase in Egs. On the other hand, inspection load increases in a piece wise linear fashion as Esg increases. At Egs level of 0.01, ATI increases by 68.75% by altering Esg from 0.01 to 0.03. However, ATI increases by 62.4% at Egs level of 0.03 for Esg varying from 0.05 to 0.10. The reason for these unusual phenomena is that higher level of AOQ forces the model to take one more cycle of inspection.

Figure 5.8 contains five curves. Each curve shows the relationship between Esg and ATI for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.05. It is observed that the inspection load decreases with the increase in Egs. On the other hand, inspection load increases in a piece wise linear fashion as Esg increases. At Egs level of 0.01, ATI increases by 67.2% by altering Esg from 0.03 to 0.05. However, ATI increases by 62.8% at Egs level of 0.03 for Esg varying from 0.05 to 0.10. The reason for these unusual phenomena is that higher level of AOQ forces the model to take one more cycle of inspection.

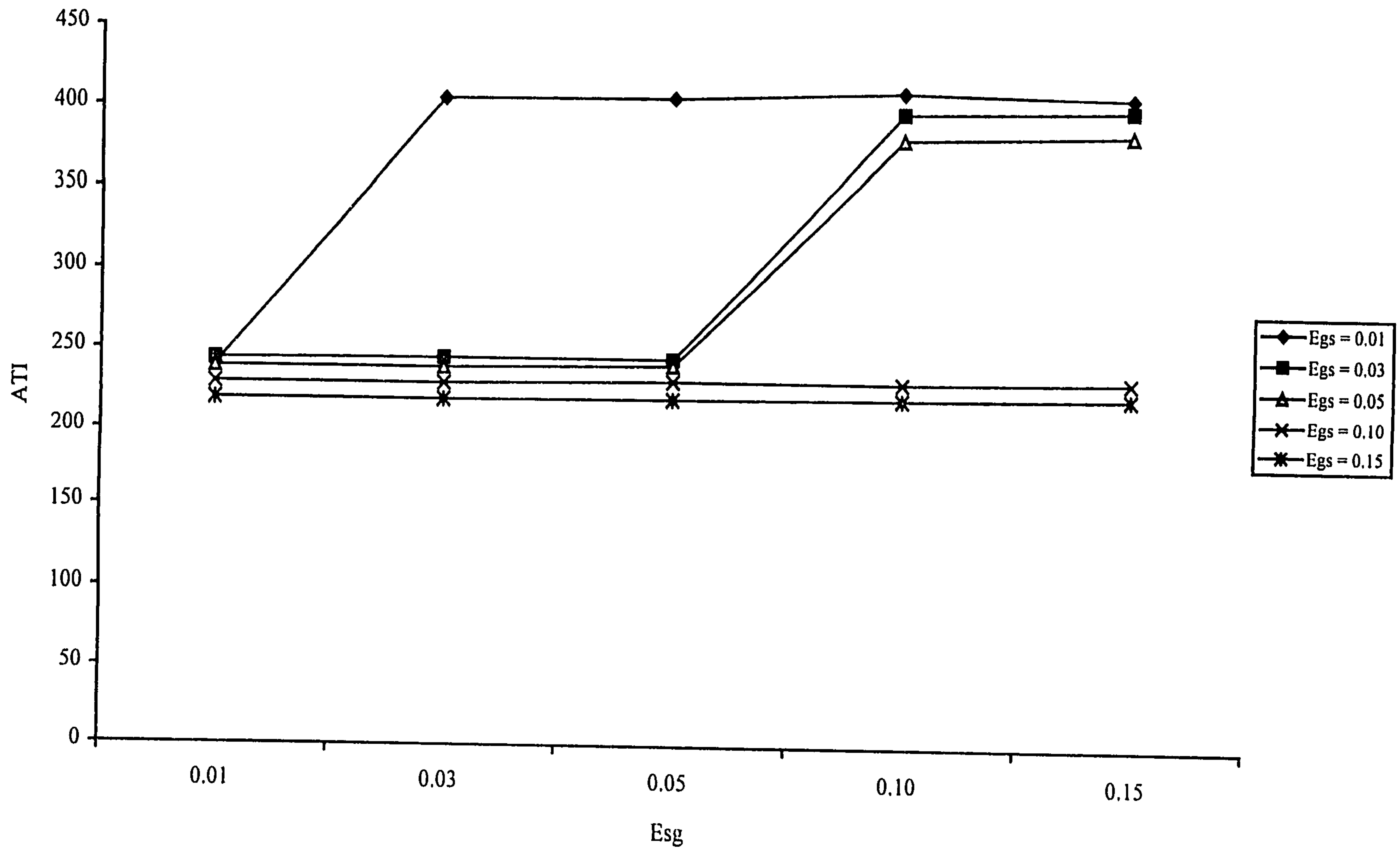

Figure 5.7 Effect of the error Esg on the Average Total Inspection (ATI) in Model 1
Egr = Erg = Ers = Esr = 0.03

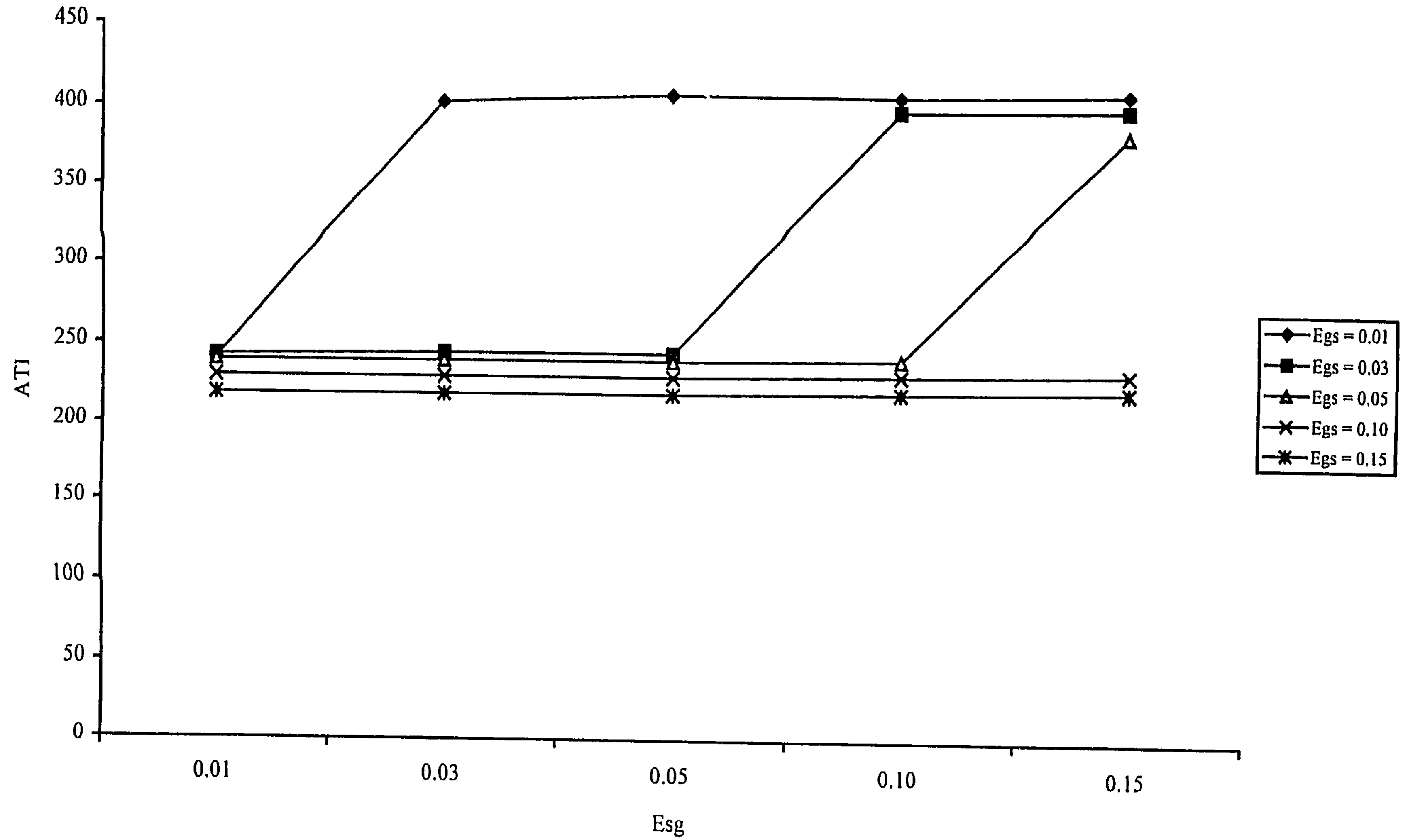

Figure 5.8 Effect of the error on the Average Total Inspection (ATI) in Model 1
Egr = Erg = Ers = Esr = 0.05

Figure 5.9 contains five curves. Each curve shows the relationship between Esg and ATI for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.10. It is observed that the inspection load decreases with the increase in Egs, i.e. the more we misclassify the good components the lesser we would have to inspect. On the other hand, inspection load increases in a piece wise linear fashion as Esg increases. At Egs level of 0.01, ATI increases by 68.3% by altering Esg from 0.03 to 0.05. However, ATI increases by 62.2% at Egs level of 0.03 for Esg varying from 0.05 to 0.10. The reason for these unusual phenomena is that higher level of AOQ forces the model to take one more cycle of inspection.

Similarly, Figure 5.10 contains five curves. Each curve shows the relationship between Esg and ATI for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.15. It is observed that the inspection load decreases with the increase in Egs while it increases with the variation in Esg from 0.01 to 0.15, which is in line with the perception. At Egs level of 0.01, ATI increases by 67% by altering Esg from 0.05 to 0.10. The reason for these unusual phenomena is that higher level of AOQ forces the model to take one more cycle of inspection.

In general it can be concluded from above description that the inspection load decreases with Egs and increases with Esg.

5.3 Economic Effects of Inspection Error on Model 2

To inspect the effects of the errors Esg (i.e. misclassifying a scrap component to be good) and Egs (i.e. misclassifying a good component to be scrap) a batch of 100 components, each with 3 characteristics was inspected with incoming quality known i.e. the joint

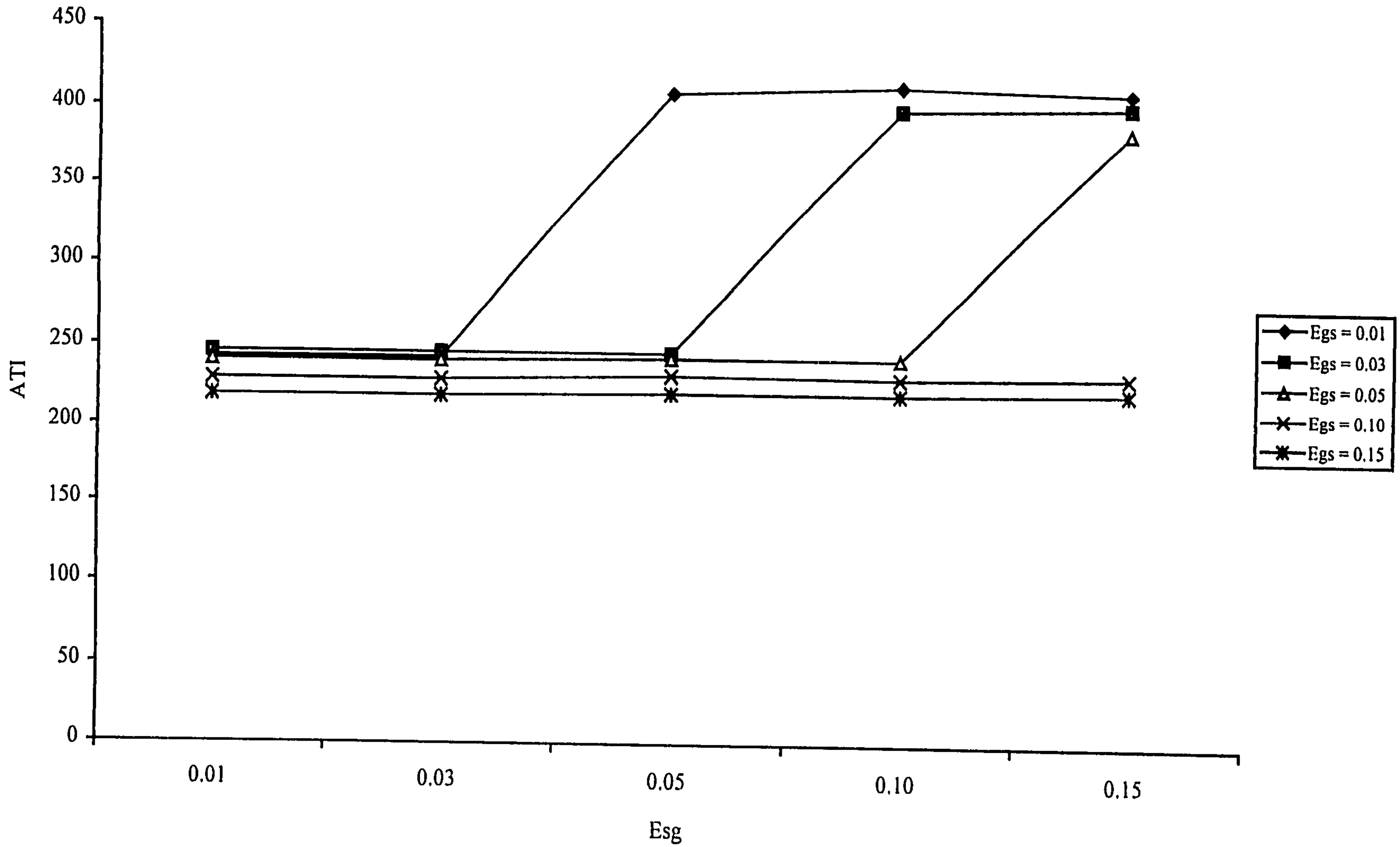

Figure 5.9 Effect of the error Esg on the Average Total Inspection (ATI) in Model 1
Egr = Erg = Ers = Esr = 0.10

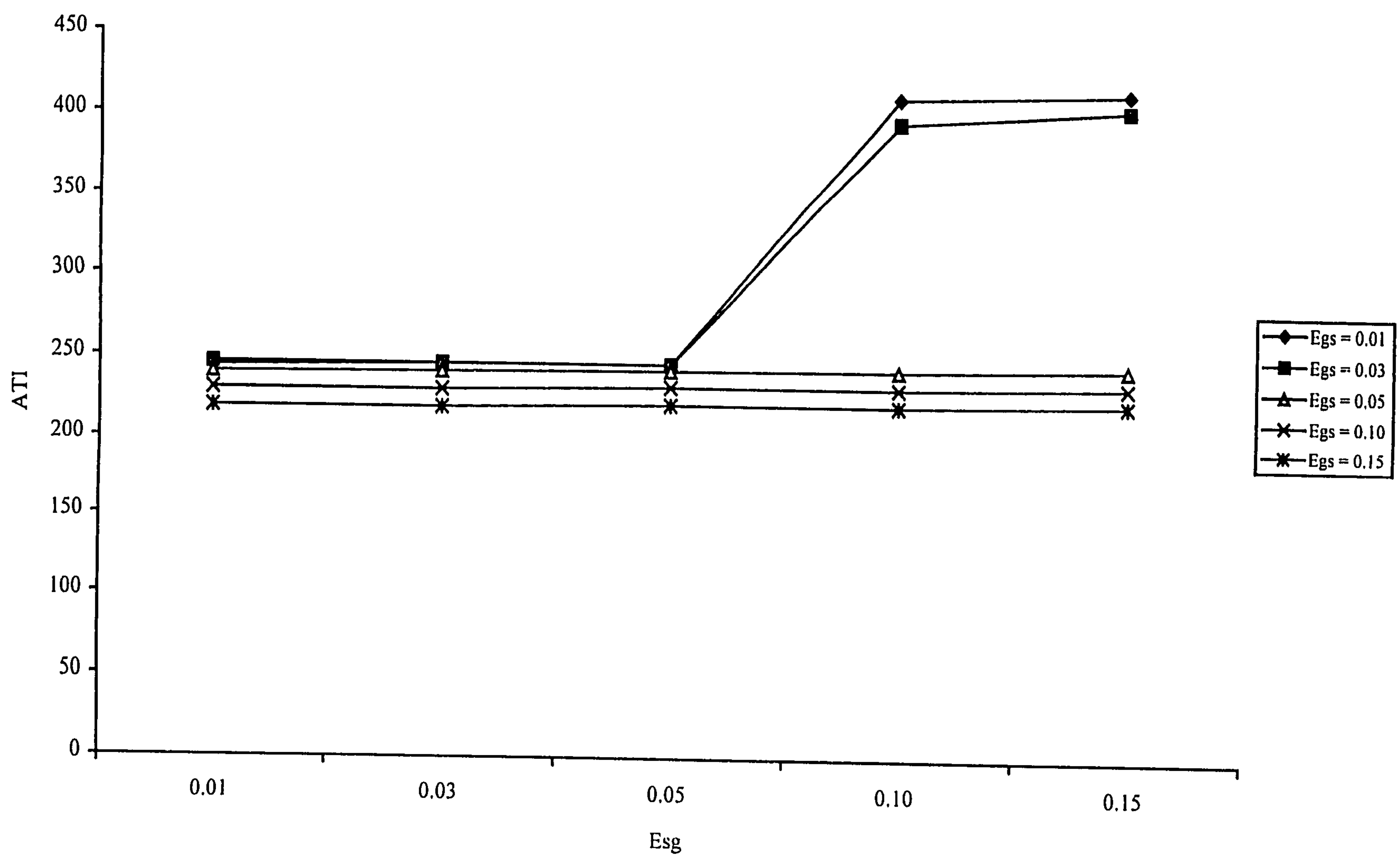

Figure 5.10 Effect of the error on the Average Total Inspection (ATI) in Model 1
Egr = Erg = Ers = Esr = 0.15

probability mass functions for the incoming components are known. These probabilities were taken to be any set of random numbers summing up to 1. The cost of inspection for the three characteristics is assumed to be 100 at the inspection station and 5000 at the rework station. The cost of false acceptance is 100000. The cost of misclassification of good components to scrap is 10000 and that of a reworkable one to scrap is 5000. The two errors of misclassifications i.e. Egs and Esg were taken to be 0.01, 0.03, 0.05, 0.10 and 0.15 to observe the effect on expected total cost per accepted component (ETC), average total inspection (ATI) and the average outgoing quality AOQ. This range of errors is obtained from [4,10]. Fixing the other errors of misclassification at any of the above five levels; the model was run 25 times each for the different values of Esg and Egs. The model determines the optimal number of repeat inspections and simulates the inspection process for each pairs of the errors. The results of the runs for the five levels of the other errors of misclassifications are shown in Table A.6 to Table A.10 in Appendix (A). Each of these tables shows the pairs of errors and optimal values for ETC, AOQ which is (1 - PG), n and A(n) for each pair of errors. Also the tables show the total cost of false rejection (TCFR), total cost of false acceptance (TCFA) and total cost of inspection (TCI). All of these costs are taken per accepted component. Figures 5.11-5.20 show the effect of the variation in Esg and Egs on AOQ and ATI at different levels of the other errors of misclassification. Next, the impact of the errors on these two performance measures of the inspection plan is examined.

5.3.1 Impact of Inspection Errors on Average Outgoing Quality

As defined earlier, we take the average outgoing quality to be the number of defective components going out of the inspection per accepted component.

Figure 5.11 contains five curves. Each curve shows the relationship between Esg and AOQ for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.01. It is observed that AOQ increases as Egs and Esg increase. For different levels of Egs the increase in AOQ is uniform when Esg varies from 0.01 to 0.15. The increase in AOQ at Egs level of 0.01 when Esg changes from 0.05 to 0.15 is 1395.53%. However, AOQ increases by 1423.53% for the same change in Esg. These enormous changes are due to the big change in Esg, i.e. 1400%. It is quite in line with intuition and indicates that model 2 is much sensitive to the error Esg. This depicts that for better quality policy, Esg should be as low as possible.

Figure 5.12 contains five curves. Each curve shows the relationship between Esg and AOQ for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.03. It is observed that AOQ increases as Egs and Esg increase. For different levels of Egs the increase in AOQ is uniform when Esg varies from 0.01 to 0.15. The increase in AOQ at Egs level of 0.01 when Esg changes from 0.05 to 0.15 is 1089.7%. However, AOQ increases by 1077.64% for the same change in Esg. These huge changes are due to the massive change in Esg, i.e. 1400%. It is quite in line with intuition and indicates that model 2 is much sensitive to the error Esg. This depicts that for better quality policy, Esg should be as low as possible.

Figure 5.13 contains five curves. Each curve shows the relationship between Esg and AOQ for a fixed level of Egs. The other errors of misclassifications are taken to be at

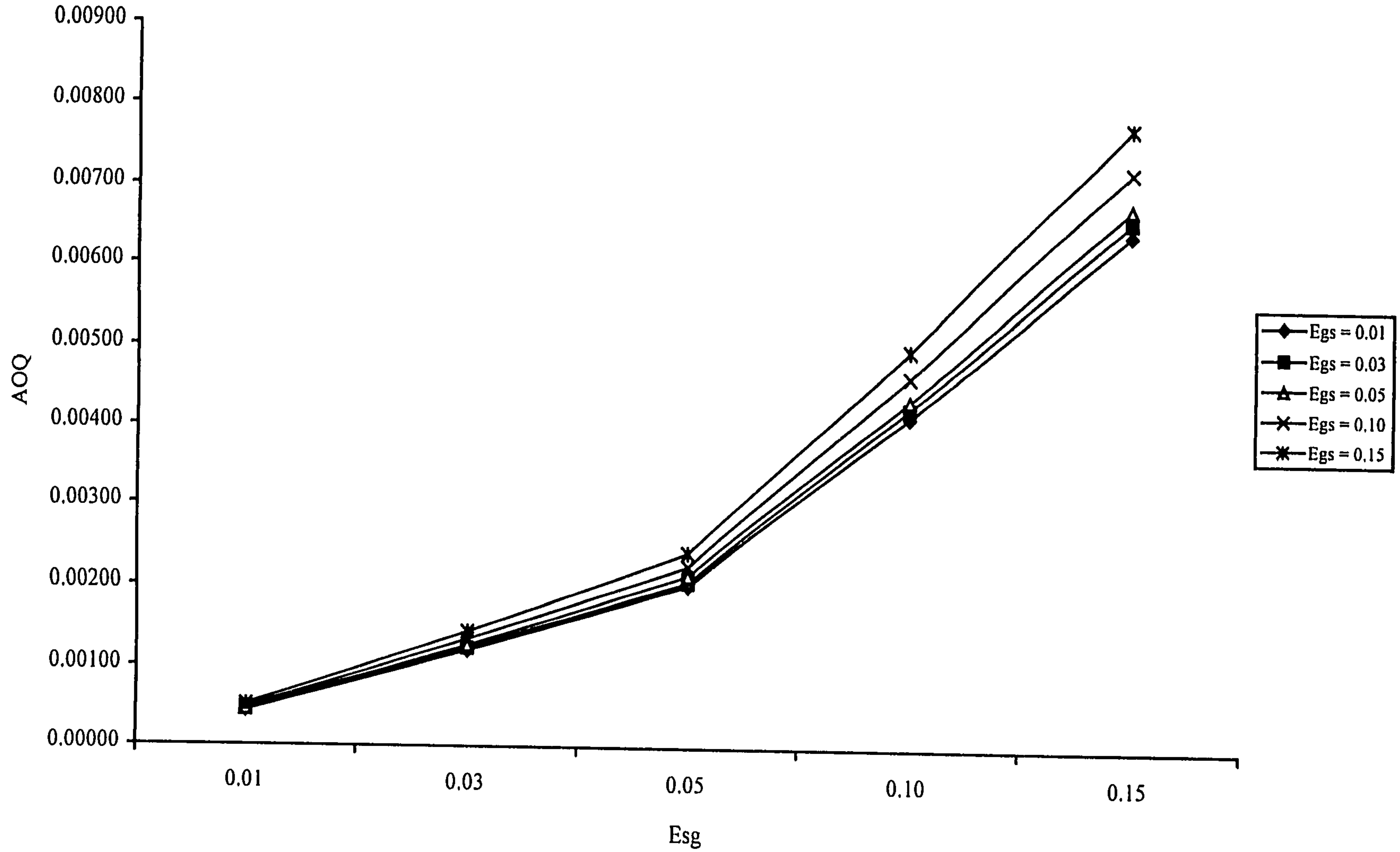

Figure 5.11 Effect of the erro r Esg on the Average Outgoing Quality (AOQ) in Model 2
Egr = Erg = Ers = Esr = 0.01

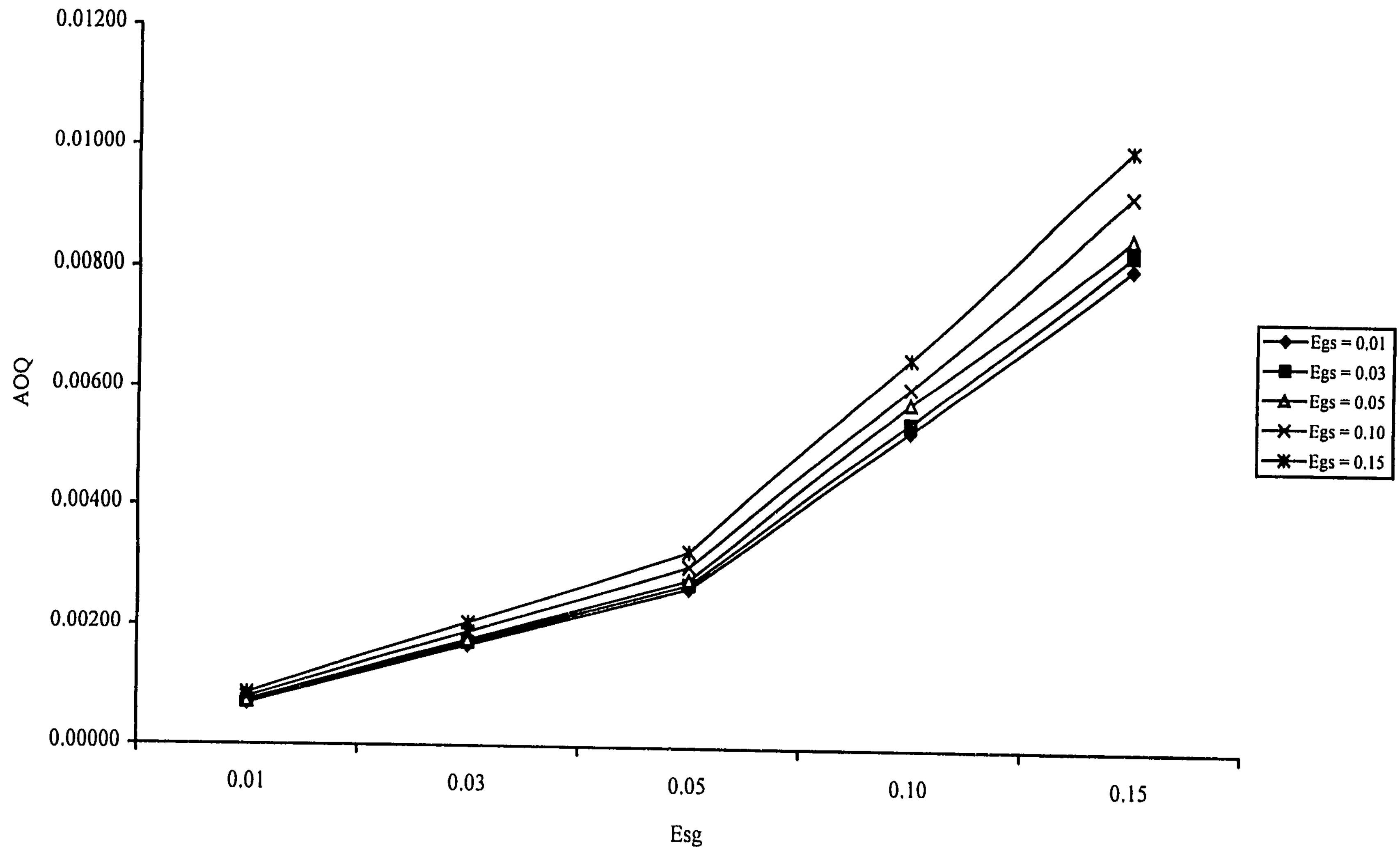

Figure 5.12 Effect of the error Esg on the Average Outgoing Quality (AOQ) in Model 2
Egr = Erg = Ers = Esr = 0.03

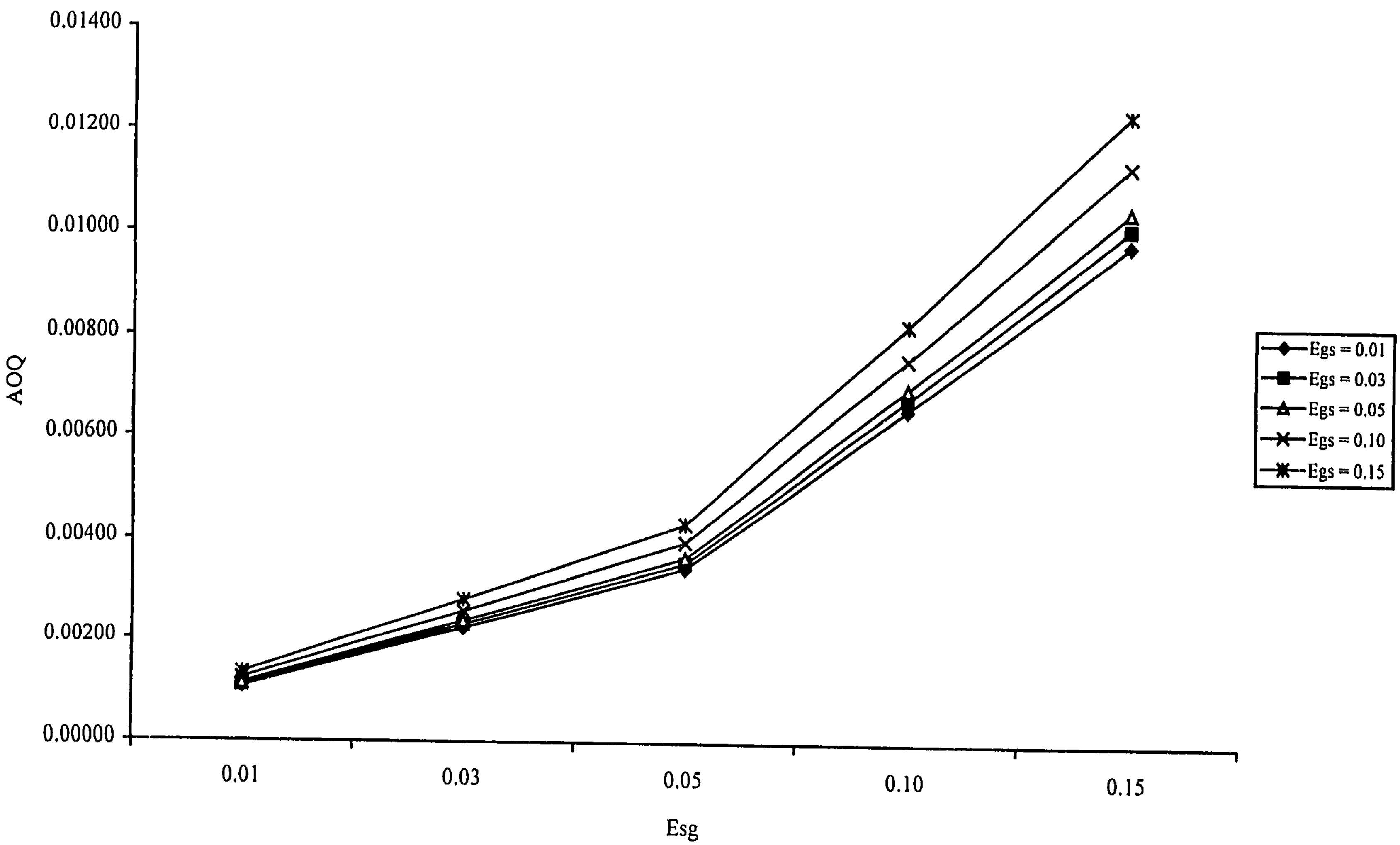

Figure 5.13 Effect of the error Esg on the Average Outgoing Quality (AOQ) in Model 2
Egr = Erg = Ers = Esr = 0.05

level 0.05. It is observed that AOQ increases as Egs and Esg increase. For different levels of Egs the increase in AOQ is uniform when Esg varies from 0.01 to 0.15. The increase in AOQ at Egs level of 0.01 when Esg changes from 0.01 to 0.15 is 846.15%. However, AOQ increases by 837.12% for the same change in Esg. These enormous changes are due to the great change in Esg, i.e. 1400%. It is quite in line with intuition and indicates that model 2 is much sensitive to the error Esg. This depicts that for better quality policy, Esg should be as low as possible.

Figure 5.14 contains five curves. Each curve shows the relationship between Esg and AOQ for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.10. It is observed that AOQ increases as Egs and Esg increase, i.e. they deteriorate the quality of the accepted components. For different levels of Egs the increase in AOQ is uniform when Esg varies from 0.01 to 0.15. The increase in AOQ at Egs level of 0.01 when Esg changes from 0.01 to 0.15 is 519.91%. However, AOQ increases by 512.38% for the same change in Esg. These enormous changes are due to the big change in Esg, i.e. 1400%. It is quite in line with intuition and indicates that model 2 is much sensitive to the error Esg. This depicts that for better quality policy, Esg should be as low as possible.

Figure 5.15 contains five curves. Each curve shows the relationship between Esg and AOQ for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.15. It is observed that AOQ increases as Egs and Esg increase, i.e. they deteriorate the quality of the accepted components. For different levels of Egs the increase in AOQ is uniform when Esg varies from 0.01 to 0.15. The increase in AOQ at Egs level of 0.01 when Esg changes from 0.01 to 0.15 is 366.59%. However, AOQ increases by 359.47% for the same change in Esg. These enormous changes are due to the massive change in

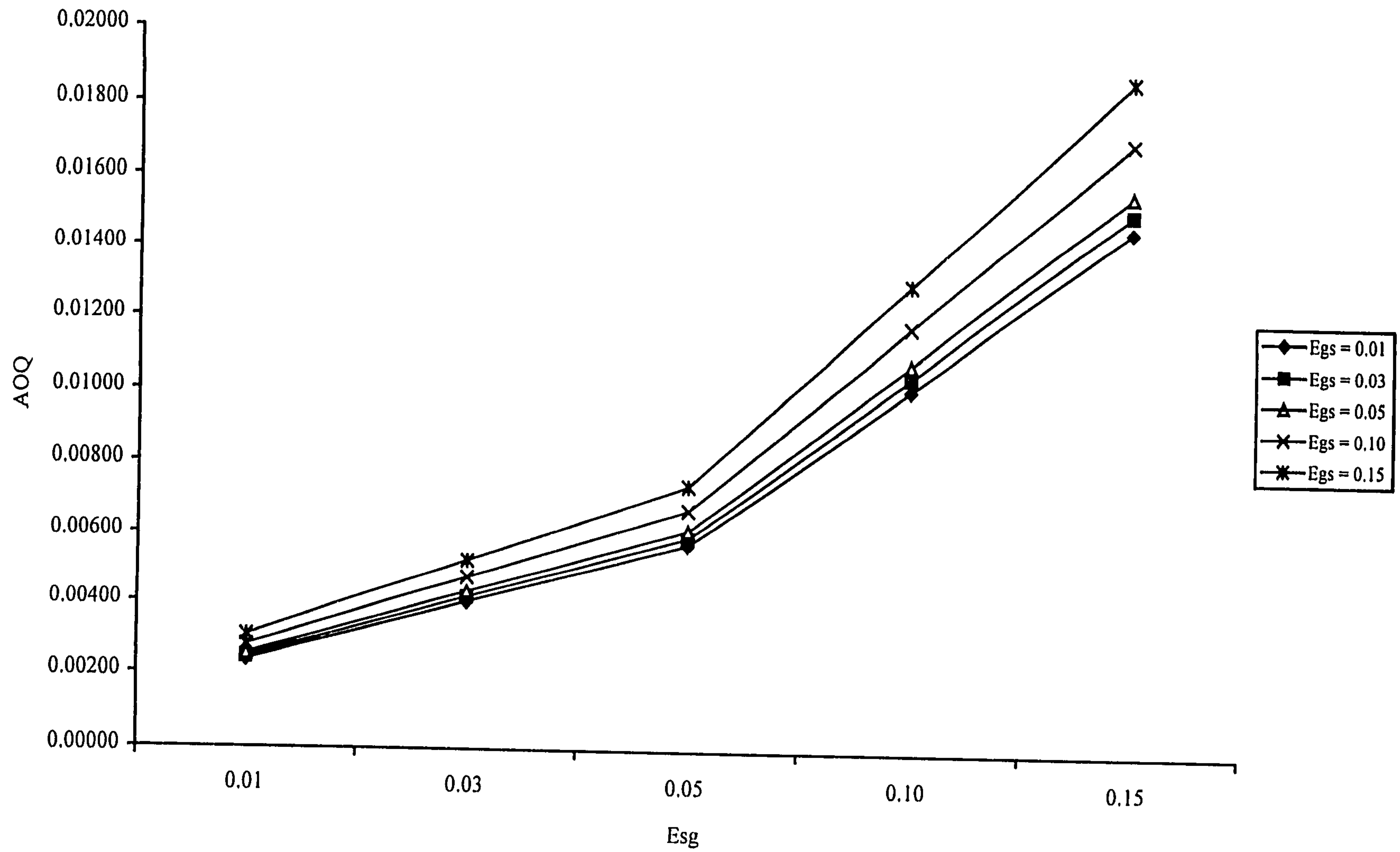

Figure 5.14 Effect of the error Esg on the Average Outgoing Quality (AOQ) in Model 2
Egr = Erg = Ers = Esr = 0.10

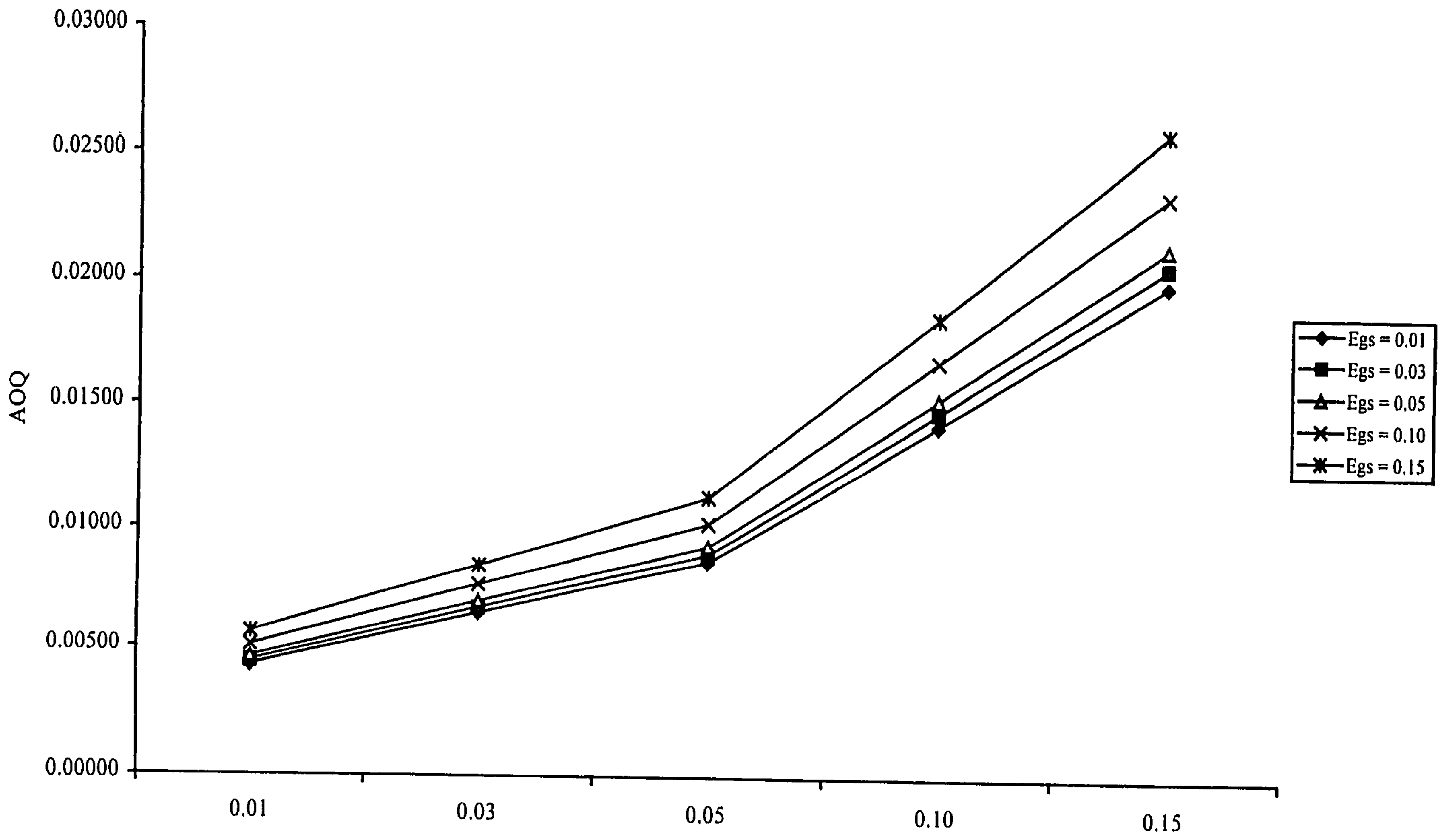

Figure 5.15 Effect of the error Esg on the Average Outgoing Quality (AOQ) in Model 2
Egr = Erg = Ers = Esr = 0.15

Esg, i.e. 1400%. It is quite in line with intuition and indicates that model 2 is much sensitive to the error Esg. This depicts that for better quality policy, Esg should be as low as possible.

In general it can be concluded that AOQ level goes up with the increase in Egs and the amount of this increase in AOQ for increasing Esg, decreases as the level of the other errors of misclassification varies from 0.01 to 0.15. It can be viewed that the other errors of misclassification increase the level of AOQ.

5.3.2 Impact of Inspection Errors on Average Total Inspection

The average total inspection is defined as the total number of inspections conducted in the optimal inspection plan. The inspection plan in the thesis is such that inspection is carried out at the characteristic-inspection-stations and at the rework stations. Thus for a

batch of M components ATI is computed as:

$$\text{ATI} = \sum_{j=1}^{n} \left(\sum_{i=1}^{N} M_{i,j} \left(\text{PG}_{i,j}\text{E}_{igr} + \text{PS}_{i,j}\text{E}_{isr} + \text{PR}_{i,j}(1 - \text{E}_{irg} - \text{E}_{irs}) \right) \right)$$

ATI can be interpreted as the inspection load in the plan.

Figure 5.16 contains five curves. Each curve shows the relationship between Esg and ATI for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.01. It is observed that the inspection load decreases with the increase in Egs and keeps increasing slightly (3.76% on the average) when Esg varies from 0.01 to 0.15 at the different levels of Egs. This is quite in line with the intuition. The increase in ATI at Egs level of 0.01 when Esg changes from 0.05 to 0.15 is 4%.

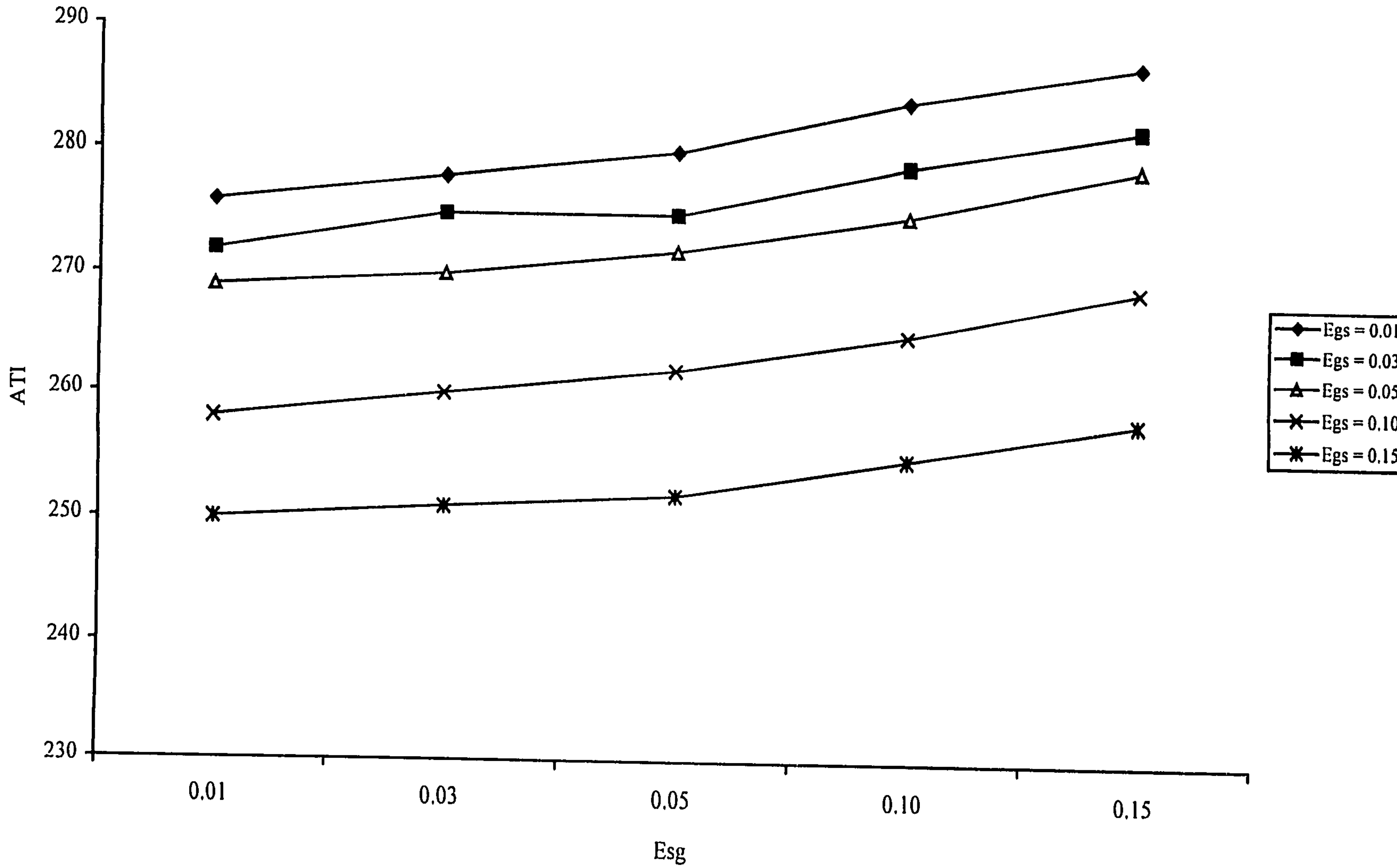

Figure 5.16 Effect of the error Esg on the Average Total Inspection (ATI) in Model 2
Egr = Erg = Ers = Esr = 0.01

Figure 5.17 contains five curves. Each curve shows the relationship between Esg and ATI for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.03. It is observed that the inspection load decreases with the increase in Egs and keeps increasing slightly (3.69% on the average) when Esg varies 0.01 to 0.15 at the different levels of Egs. This is quite in line with the intuition. The increase in ATI at Egs level of 0.01 when Esg changes from 0.05 to 0.15 is 3.24%.

Figure 5.18 contains five curves. Each curve shows the relationship between Esg and ATI for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.05. It is observed that the inspection load decreases with the increase in Egs and keeps increasing slightly (3.52% on the average) when Esg varies 0.01 to 0.15 at the different levels of Egs. This is quite in line with the intuition. The increase in ATI at Egs level of 0.01 when Esg changes from 0.05 to 0.15 is 3.58%.

Figure 5.19 contains five curves. Each curve shows the relationship between Esg and ATI for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.10. It is observed that the inspection load decreases with the increase in Egs and keeps increasing slightly (3.70% on the average) when Esg varies 0.01 to 0.15 at the different levels of Egs. This is quite in line with the intuition. The increase in ATI at Egs level of 0.01 when Esg changes from 0.05 to 0.15 is 3.56%.

Similarly, figure 5.20 contains five curves. Each curve shows the relationship between Esg and ATI for a fixed level of Egs. The other errors of misclassifications are taken to be at level 0.01. It is observed that the inspection load decreases with the increase in Egs and keeps increasing slightly (3.29% on the average) when Esg varies 0.01 to 0.15 at the

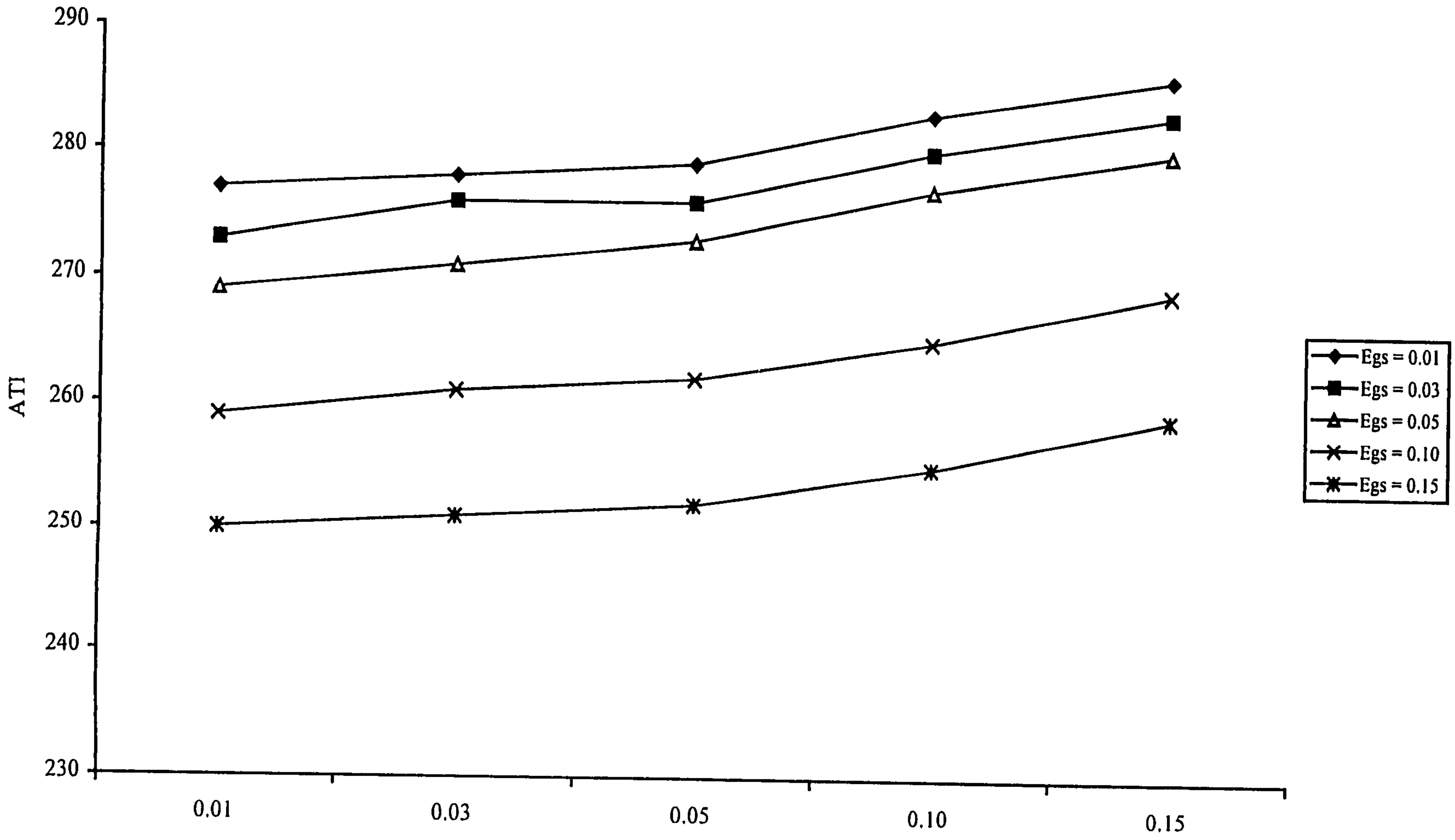

Figure 5.17 Effect of the error Esg on the Average Total Inspection (ATI) in Model 2
Egr = Erg = Ers = Esr = 0.03

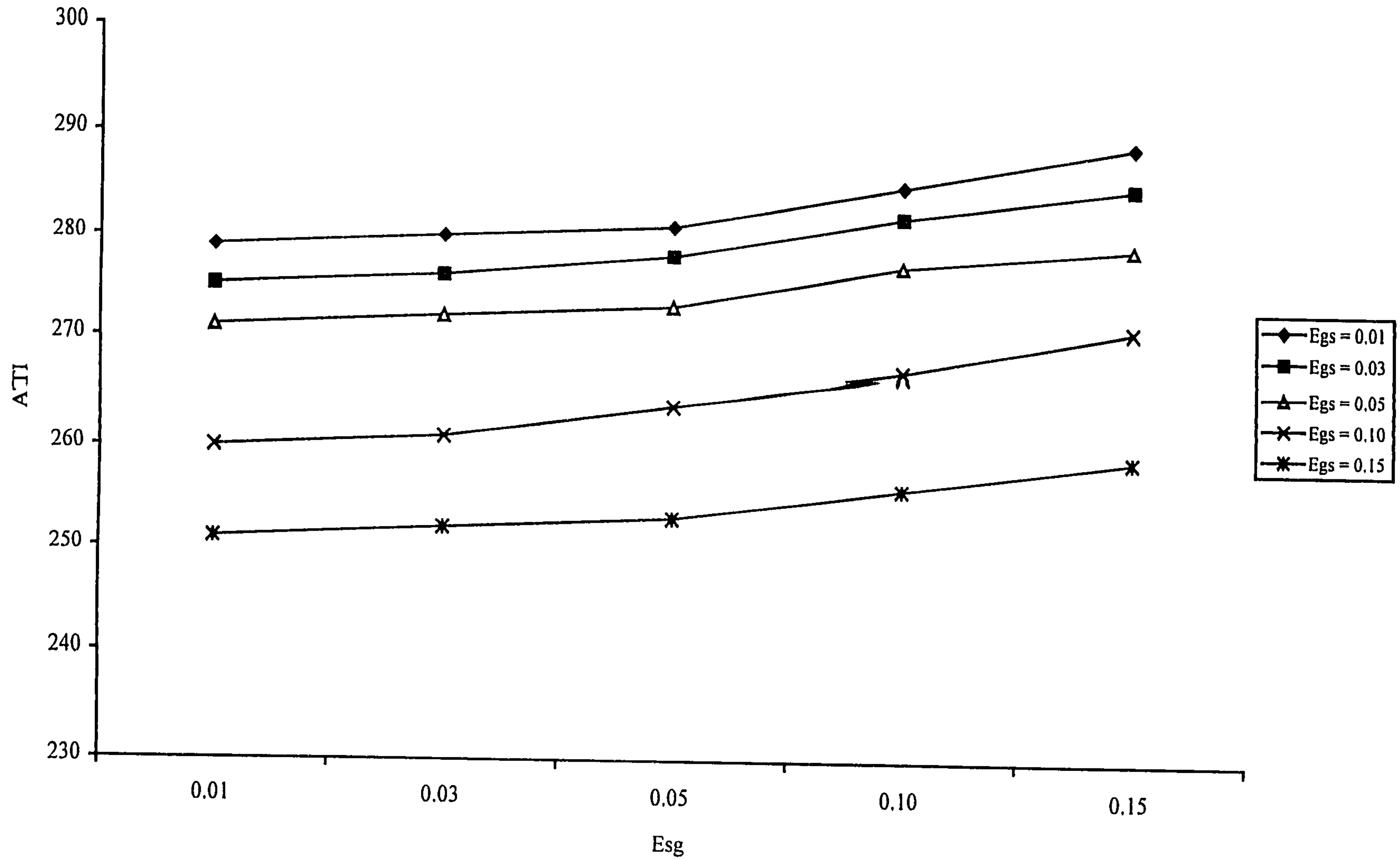

Figure 5.18 Effect of the error Esg on the Average Total Inspection (ATI) in Model 2
Egr = Erg = Ers = Esr = 0.05

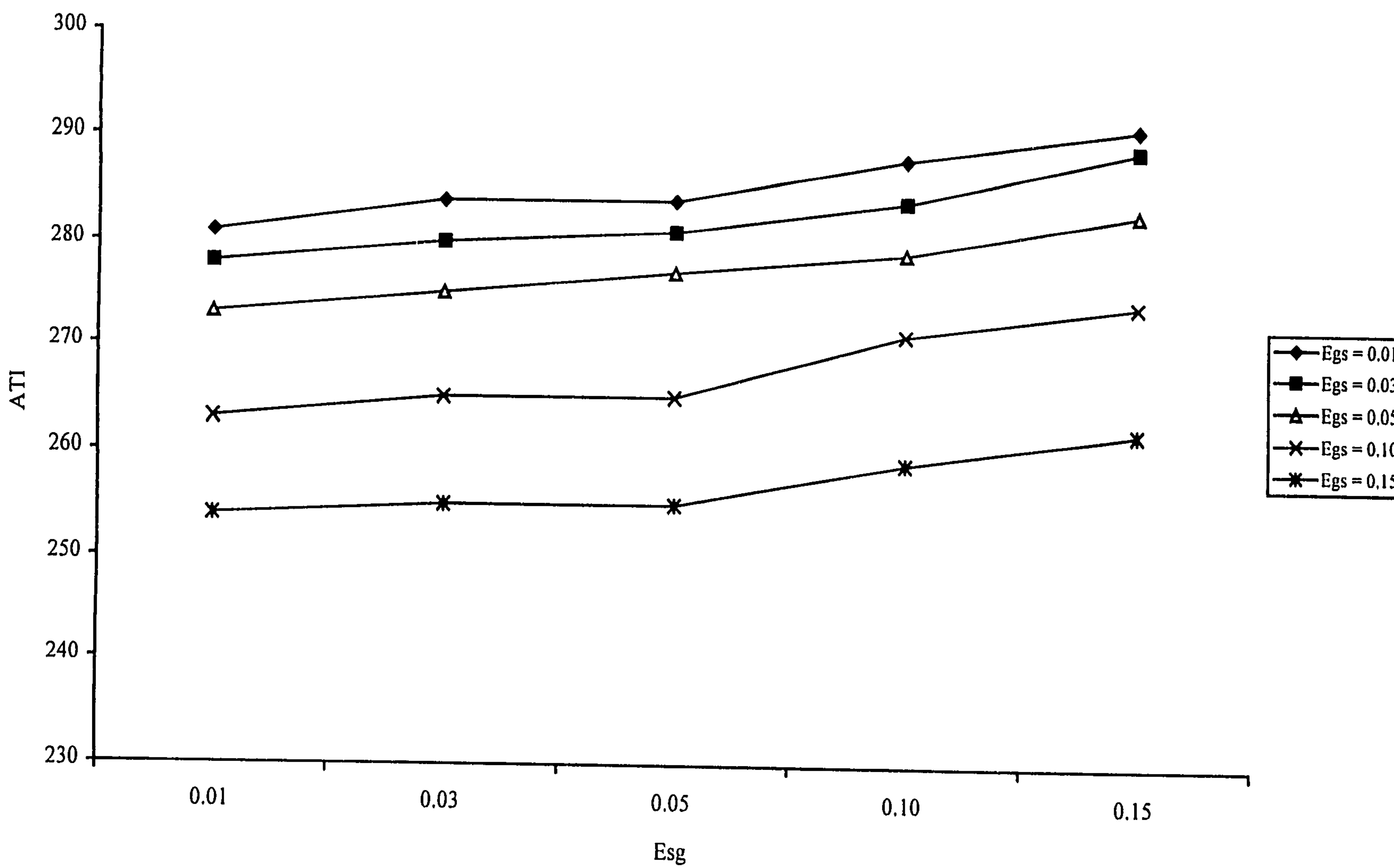

Figure 5.19 Effect of the error Esg on the Average Total Inspection (ATI) in Model 2
Egr = Erg = Ers = Esr = 0.10

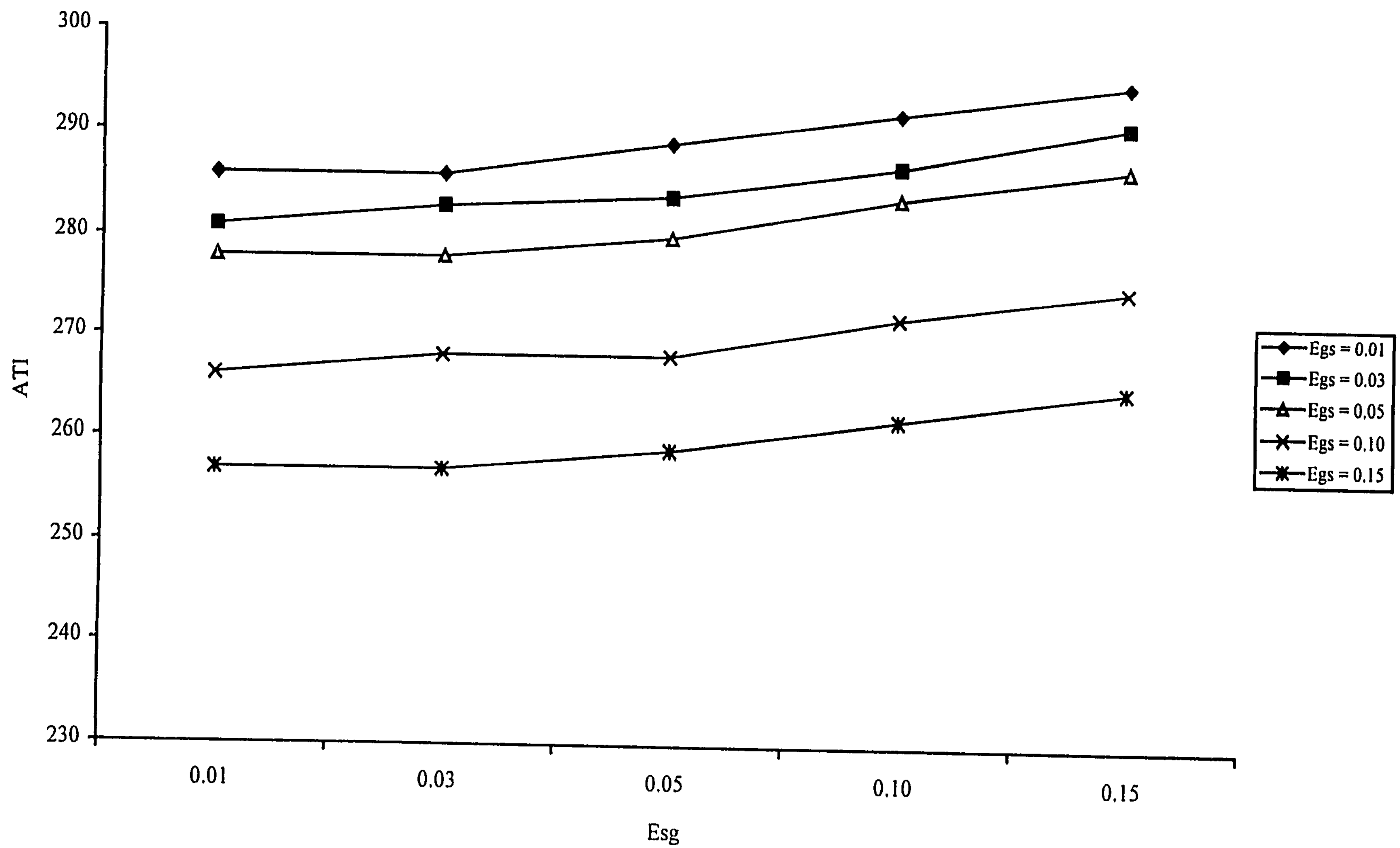

Figure 5.20 Effect of the error Esg on the Average Total Inspection (ATI) in Model 2
Egr = Erg = Ers = Esr = 0.15

different levels of Egs. This is quite in line with the intuition. The increase in ATI at Egs level of 0.01 when Esg changes from 0.05 to 0.15 is 3.14%.

In general it can be concluded that ATI decreases with Egs and increases slightly with Esg at the different levels of other misclassifications.

5.4 Analysis of the Results

The inspection plan studied in the thesis contains various forms of type I and type II errors. The study is made by varying Esg and Egs within a certain range and keeping the other errors misclassifications fixed at a certain level.

As indicated in Tables A.1-A.5 the expected total cost of inspection for the 1st model in the thesis keeps on increasing when other errors of misclassifications are fixed and Esg varies from 0.01 to 0.15 and Egs remains at a certain level. The amount of this increase in ETC goes up with the fixed level of Egs. It is observed that there occurs a drastic increase of 224.12% (on the average) when both Esg and Egs change from 0.01 to 0.15. Similarly, the Tables A.6-A.10 indicate that expected total cost of inspection for the 2nd model keeps on increasing when other errors of misclassifications are fixed and Esg varies from 0.01 to 0.15 and Egs remains at a certain level. The amount of this increase in ETC goes up with the fixed level of Egs. It is observed that there occurs a drastic increase of 224.12% (on the average) when both Esg and Egs change from 0.01 to 0.15.

As indicated in Tables A.1-A.5 and Figures 5.1-5.5 the average outgoing quality for the 1st model in the thesis keeps on increasing when other errors of misclassifications are fixed and Esg varies from 0.01 to 0.15 and Egs remains at a certain level. The same pattern is observed at the different levels of other errors of misclassifications. The amount of increase in AOQ goes up with these misclassifications. Similarly, the Tables A.6-A.10 and Figures 5.11-5.15 indicate that average outgoing quality for the 2nd model keeps on increasing when other errors of misclassifications are fixed and Esg varies from 0.01 to 0.15 and Egs remains at a certain level. The amount of this increase in AOQ goes up with the fixed level of Egs. The same pattern is observed at the different levels of other errors of misclassifications. The amount of increase in AOQ goes up with these misclassifications. There are some abnormal trends in the variation of AOQ in model 1. For example AOQ falls rapidly when Esg varies from 0.05 to 0.10 at Egs fixed at 0.03 or 0.05 and other errors of misclassifications at any of the five levels.

As indicated in tables A.1-A.5 and Figures 5.6-5.10 the average total inspection for the 1st model in the thesis keeps on increasing when other errors of misclassifications are fixed and Esg varies from 0.01 to 0.15 and Egs remains at a certain level. The same pattern is observed at the different levels of other errors of misclassifications. The overall decrease in ATI with the increase in the level of Egs does not deviate much with the variation in the other errors of misclassification. Similarly, the table A.6-A.10 and Figures 5.16-5.20 indicate that ATI for the 2nd model does not change much when other errors of misclassifications are fixed and Esg varies from 0.01 to 0.15 and Egs remains at a certain level. The same pattern is observed at the different levels of other errors of misclassification. There are some abnormal trends in the variation of ATI in model 1. For example ATI goes up rapidly when Esg varies from 0.05 to 0.10 at Egs fixed at 0.03 or 0.05 and other errors of misclassifications at any of the five levels.

The impact of the higher values of Esg on the two models at a fixed level of other misclassifications can be summarized as:

- An increased inspection load.
- A higher expected total cost of inspection.
- A higher value of the average outgoing quality.

On the other hand, the impact of the higher values of Egs on the two models at a fixed level of other misclassifications can be summarized as:

- A reduced inspection load.
- A higher expected total cost of inspection.
- A higher value of the average outgoing quality.

Similarly, we can summarize the impact of the other misclassifications at a certain level of both Esg and Egs as:

- A higher total cost of inspection.
- A higher value of average outgoing quality.

5.5 Conclusion

The purpose of the chapter was to quantify the effect of the inspection errors, mainly Esg and Egs on the performance measures of the complete repeat inspection plan presented in the thesis. This is accomplished by conducting sensitivity analysis on the errors and then observing the changes in ETC, ATI and AOQ. As a result of this analysis, the following could be concluded for the two models developed in the thesis:

- The errors Esg and Egs both increase ETC, but Egs increases ETC at a faster rate especially at values ≥ 0.05.
- AOQ is higher for higher values of Esg and Egs both.

- ATI decreases as Egs increases. This is in line with intuition , since an increase in the probability of rejecting good components will reduce the number will reduce the number of components to be inspected in subsequent cycles of inspection. On the other hand as Esg increases ATI increases. Therefore higher values of Esg increases inspection load in the plan.

It can be resolved that Egs and Esg have significant effect on the measures of performance of the repeat inspection plan studied in the thesis.

CHAPTER 6

CONCLUSIONS

6.1 Summary and Conclusions

The objective of the research was to extend the repeat multicharacteristic inspection plan in the literature. The background of these inspection plans is the fact that most of the inspection processes are error prone and the assumption of perfect inspection is not valid. The plans in the literature assume only type I and type II errors made by the inspector. The work in this research was motivated by the realization of a need to a new inspection plan which allows a very general classification of the product to be made by the inspector, i.e. non-defective, to be reworked or to be scrapped. This gives rise to several forms of inspection error that an inspector can commit. The extensions of the research were based on a complete inspection plan given by Raouf [21]. The plan is established to guard against inspection errors. The implications of these errors could be detrimental in the event of a critical component failure.

The new inspection plan for multicharacteristic repeat inspection of critical components was described in chapter 3 of the thesis. A model to depict the proposed plan was developed and described. Computational procedure was suggested and a software implementing it was developed. A numerical example was presented to describe the procedure of obtaining the optimal number of repeat inspections.

The practicality of the above model was enhanced in chapter 4 of the thesis by modifying it for the case where characteristics' defective rates are statistically dependent. This requires by using the knowledge of the joint probability mass function (j.p.m.f.) of the random variables representing the characteristic defective rates. Rules of updating the quality of the dependent characteristics were proposed which were consistent with the basic probability rules. The complete model was developed and described. Computational procedure was suggested and a software implementing it was developed. A numerical example was presented to describe the procedure of obtaining the optimal number of repeat inspections. To validate the mathematical modeling, the two models were tested on an independent and a dependent problem. The two models performed identically for the independent problem in terms of the optimal number of inspection cycles, average outgoing quality and the expected total cost. Whereas in the case of dependent problem, model 2 performs better in terms of all the above performance measures. This is exactly in line with the intuition.

The sensitivity analysis of the inspection errors on the two models developed was carried out in chapter 5 of the thesis by observing the behaviors of ETC, AOQ and ATI. A detailed study was made from the various combinations of Egs and Esg by fixing the other errors of misclassification at different levels. The results were in line with the intuition and were found to be consistent with ones in the literature.

The results of the research can be applied to many practical situations where the inspection process characterizes the product into three general classes and where the cost of committing inspection errors is unavoidable.

6.2 Future Research

The research in this thesis presented a new complete inspection plan for critical components. Two models were developed to depict the plan. Some of the suggestions regarding the possible research directions in the future are as follows:

1. Inspection plan in the thesis can be extended under varying inspection errors. In this extension, the error can be made a function of incoming quality.
2. The approach in Al-Najjar [2] can be used to extend the two models in this thesis by relaxing the assumption of equality of the number of inspections and by allowing variable number of inspections for different characteristics. Then, a sensitivity analysis can also be carried out for these models.
3. The inspection plan studied in the thesis can be generalized by incorporating the material handling cost.
4. A more detailed study of the errors could be conducted.

APPENDIX (A)

STUDY OF THE EFFECTS OF INSPECTION ERROR ON THE TWO MODELS

Table A.1 Effect of the Inspection Errors on the Inspection Plan of Model 1
Egr = Erg = Ers = Esr = 0.01

No.	Egs	Esg	1-PG	A(n)	ATI	TCFA	TCFR	TCI	ETC	n
1	0.01	0.01	0.00615	55	241	611.56	340.35	3609.21	4561.11	1
2	0.01	0.03	0.00036	52	403	36.25	668.27	4309.26	5013.78	2
3	0.01	0.05	0.00094	52	405	96.25	669.52	4319.98	5085.75	2
4	0.01	0.10	0.00370	52	409	376.84	670.38	4350.06	5397.28	2
5	0.01	0.15	0.00827	53	414	841.98	660.11	4300.38	5802.47	2
6	0.03	0.01	0.00627	52	244	670.94	994.76	4132.01	5797.70	1
7	0.03	0.03	0.01376	52	244	1480.96	992.00	4131.97	6604.93	1
8	0.03	0.05	0.02116	53	246	2276.81	979.33	4082.21	7338.35	1
9	0.03	0.10	0.00385	45	396	394.00	2121.53	5334.88	7850.41	2
10	0.03	0.15	0.00861	46	401	879.73	2084.61	5261.49	8225.83	2
11	0.05	0.01	0.00640	48	239	694.18	1730.01	4400.90	6825.08	1
12	0.05	0.03	0.01404	49	241	1524.35	1705.54	4341.58	7571.48	1
13	0.05	0.05	0.02160	49	241	2353.33	1700.71	4341.55	8395.58	1
14	0.05	0.10	0.00402	41	385	413.23	3725.52	5773.40	9912.16	2
15	0.05	0.15	0.00897	41	389	923.42	3731.14	5817.46	10472.02	2
16	0.10	0.01	0.00676	41	228	723.47	3798.54	4960.10	9482.10	1
17	0.10	0.03	0.01482	42	230	1585.88	3733.46	4877.60	10196.93	1
18	0.10	0.05	0.02278	42	230	2447.90	3722.56	4877.55	11048.01	1
19	0.10	0.10	0.04231	43	232	4551.26	3644.53	4798.83	12994.61	1
20	0.10	0.15	0.06129	44	234	6598.48	3569.79	4723.68	14891.95	1
21	0.15	0.01	0.00715	34	218	780.19	6505.10	5768.34	13053.63	1
22	0.15	0.03	0.01568	35	220	1704.95	6365.26	5646.26	13716.47	1
23	0.15	0.05	0.02410	35	220	2631.24	6346.19	5646.21	14623.64	1
24	0.15	0.10	0.04472	36	222	4876.90	6186.32	5530.83	16594.05	1
25	0.15	0.15	0.06473	37	224	7049.83	6034.58	5421.69	18506.11	1

Table A.2 Effect of the Inspection Errors on the Inspection Plan of Model 1
Egr = Erg = Ers = Esr = 0.03

No.	Egs	Esg	1-PG	A(n)	ATI	TCFA	TCFR	TCI	ETC	n
1	0.01	0.01	0.01100	54	240	1073.43	405.42	3934.56	5413.41	1
2	0.01	0.03	0.00055	52	405	55.10	734.00	4961.52	5750.61	2
3	0.01	0.05	0.00114	51	406	113.99	747.83	5068.82	5930.65	2
4	0.01	0.10	0.00389	52	411	389.27	736.63	5013.62	6139.52	2
5	0.01	0.15	0.00846	51	409	846.32	740.79	5100.79	6687.91	2
6	0.03	0.01	0.01122	51	244	1188.61	1080.85	4481.70	6751.16	1
7	0.03	0.03	0.01866	52	245	1980.02	1062.19	4409.26	7451.48	1
8	0.03	0.05	0.02601	52	245	2772.23	1059.46	4409.17	8240.86	1
9	0.03	0.10	0.00405	45	398	406.84	2202.17	6041.13	8650.14	2
10	0.03	0.15	0.00881	45	401	884.10	2199.18	6083.12	9166.39	2
11	0.05	0.01	0.01146	48	239	1204.97	1795.40	4679.25	7679.62	1
12	0.05	0.03	0.01905	48	239	2015.91	1790.55	4679.15	8485.61	1
13	0.05	0.05	0.02655	49	241	2808.57	1765.36	4616.98	9190.90	1
14	0.05	0.10	0.00422	40	382	427.25	3849.20	6624.25	10900.71	2
15	0.05	0.15	0.00918	40	386	928.34	3856.30	6678.42	11463.06	2
16	0.10	0.01	0.01209	41	229	1274.85	3879.79	5285.15	10439.79	1
17	0.10	0.03	0.02009	41	229	2132.33	3868.74	5285.03	11286.10	1
18	0.10	0.05	0.02800	42	231	2964.77	3802.47	5198.10	11965.34	1
19	0.10	0.10	0.04739	42	231	5059.78	3775.70	5197.81	14033.28	1
20	0.10	0.15	0.06624	43	233	7078.81	3697.17	5114.75	15890.73	1
21	0.15	0.01	0.01280	34	219	1373.69	6597.07	6140.39	14111.15	1
22	0.15	0.03	0.02125	35	219	2231.44	6389.73	5964.80	14585.98	1
23	0.15	0.05	0.02961	35	220	3122.80	6402.14	5990.89	15515.83	1
24	0.15	0.10	0.05008	36	221	5280.63	6211.99	5844.19	17336.80	1
25	0.15	0.15	0.06994	37	223	7370.54	6061.00	5730.22	19161.76	1

Table A.3 Effect of the Inspection Errors on the Inspection Plan of Model 1
Egr = Erg = Ers = Esr = 0.05

No.	Egs	Esg	1-PG	A(n)	ATI	TCFA	TCFR	TCI	ETC	n
1	0.01	0.01	0.01582	54	241	1517.24	468.41	4213.07	6198.72	1
2	0.01	0.03	0.00096	50	403	92.55	826.83	5787.12	6706.50	2
3	0.01	0.05	0.00152	51	408	150.23	818.23	5726.75	6695.20	2
4	0.01	0.10	0.00427	51	408	420.28	812.01	5725.70	6958.00	2
5	0.01	0.15	0.00884	50	411	885.74	827.84	5878.65	7592.23	2
6	0.03	0.01	0.01614	50	243	1698.83	1165.86	4819.86	7684.55	1
7	0.03	0.03	0.02353	51	245	2477.86	1150.34	4760.03	8388.23	1
8	0.03	0.05	0.03083	51	245	3260.26	1147.58	4759.89	9167.73	1
9	0.03	0.10	0.00445	45	399	438.92	2280.33	6720.41	9439.65	2
10	0.03	0.15	0.00920	45	401	906.77	2272.55	6746.02	9925.33	2
11	0.05	0.01	0.01648	48	240	1713.98	1870.14	4968.08	8552.19	1
12	0.05	0.03	0.02401	48	240	2511.06	1865.30	4967.92	9344.29	1
13	0.05	0.05	0.03147	48	240	3303.69	1860.50	4967.77	10131.97	1
14	0.05	0.10	0.04976	49	242	5240.58	1827.33	4902.27	11970.18	1
15	0.05	0.15	0.00959	39	385	979.39	4007.09	7598.48	12584.95	2
16	0.10	0.01	0.01738	41	230	1812.12	3962.12	5606.94	11381.17	1
17	0.10	0.03	0.02533	41	230	2654.12	3951.09	5606.75	12211.96	1
18	0.10	0.05	0.03318	41	230	3491.18	3940.16	5606.57	13037.91	1
19	0.10	0.10	0.05243	42	232	5525.39	3856.74	5514.97	14897.10	1
20	0.10	0.15	0.07114	43	234	7505.05	3776.94	5427.63	16709.62	1
21	0.15	0.01	0.01839	34	219	1912.84	6654.57	6485.05	15052.46	1
22	0.15	0.03	0.02679	34	219	2800.81	6635.42	6484.82	15921.05	1
23	0.15	0.05	0.03508	34	219	3683.25	6616.45	6484.60	16784.30	1
24	0.15	0.10	0.05539	35	221	5812.08	6447.05	6349.65	18608.78	1
25	0.15	0.15	0.07511	36	223	7872.77	6286.51	6222.19	20381.47	1

Table A.4 Effect of the Inspection Errors on the Inspection Plan of Model 1
Egr = Erg = Ers = Esr = 0.10

No.	Egs	Esg	1-PG	A(n)	ATI	TCFA	TCFR	TCI	ETC	n
1	0.01	0.01	0.02773	54	243	2554.19	627.74	4874.30	8056.23	1
2	0.01	0.03	0.03485	54	243	3223.95	626.72	4874.03	8724.69	1
3	0.01	0.05	0.00341	50	409	315.60	1017.23	7398.51	8731.33	2
4	0.01	0.10	0.00607	50	414	583.21	1022.44	7475.42	9081.08	2
5	0.01	0.15	0.01062	48	411	1018.96	1051.79	7745.17	9815.92	2
6	0.03	0.01	0.02829	51	246	2850.67	1326.29	5402.98	9579.94	1
7	0.03	0.03	0.03555	51	246	3592.89	1323.54	5402.75	10319.18	1
8	0.03	0.05	0.04274	51	246	4331.15	1320.82	5402.52	11054.48	1
9	0.03	0.10	0.00647	43	399	620.43	2585.88	8736.89	11943.20	2
10	0.03	0.15	0.01106	43	402	1079.18	2584.73	8799.97	12463.87	2
11	0.05	0.01	0.02887	47	241	2946.55	2086.14	5753.48	10786.16	1
12	0.05	0.03	0.03628	47	241	3713.26	2081.28	5753.22	11547.77	1
13	0.05	0.05	0.04361	48	243	4453.26	2051.44	5677.19	12181.88	1
14	0.05	0.10	0.06160	49	243	6202.95	1998.02	5560.73	13761.69	1
15	0.05	0.15	0.01177	38	386	1136.16	4318.59	9635.46	15090.21	2
16	0.10	0.01	0.03044	40	229	3060.16	4215.02	6479.41	13754.59	1
17	0.10	0.03	0.03824	40	229	3855.15	4204.07	6479.11	14538.33	1
18	0.10	0.05	0.04596	41	232	4614.69	4130.40	6372.46	15117.55	1
19	0.10	0.10	0.06487	41	231	6558.26	4103.88	6371.73	17033.86	1
20	0.10	0.15	0.08326	42	233	8420.74	4018.51	6269.74	18708.99	1
21	0.15	0.01	0.03219	33	219	3291.19	7029.19	7542.86	17863.25	1
22	0.15	0.03	0.04043	33	219	4144.66	7009.94	7542.49	18697.09	1
23	0.15	0.05	0.04857	34	221	4945.04	6854.64	7382.70	19182.38	1
24	0.15	0.10	0.06850	34	221	7023.58	6808.25	7381.79	21213.63	1
25	0.15	0.15	0.08786	35	223	8988.75	6635.50	7230.62	22854.87	1

Table A.5 Effect of the Inspection Errors on the Inspection Plan of Model 1
Egr = Erg = Ers = Esr = 0.15

No.	Egs	Esg	1-PG	A(n)	ATI	TCFA	TCFR	TCI	ETC	n
1	0.01	0.01	0.03944	53	244	3496.65	804.28	5594.25	9895.18	1
2	0.01	0.03	0.04645	54	246	4121.08	794.43	5533.39	10448.91	1
3	0.01	0.05	0.05338	54	246	4748.39	793.39	5533.07	11074.85	1
4	0.01	0.10	0.00928	47	411	835.64	1295.86	9461.98	11593.48	2
5	0.01	0.15	0.01358	48	415	1246.24	1271.92	9342.19	11860.35	2
6	0.03	0.01	0.04023	49	246	3996.97	1556.98	6237.79	11791.75	1
7	0.03	0.03	0.04737	50	246	4692.46	1529.51	6136.15	12358.11	1
8	0.03	0.05	0.05444	50	246	5402.00	1526.75	6135.88	13064.63	1
9	0.03	0.10	0.00966	41	396	901.89	2928.71	10762.34	14592.94	2
10	0.03	0.15	0.01447	43	405	1335.66	2828.80	10445.79	14610.26	2
11	0.05	0.01	0.04105	46	240	4046.69	2304.17	6515.01	12865.86	1
12	0.05	0.03	0.04834	47	241	4747.69	2260.55	6401.00	13409.23	1
13	0.05	0.05	0.05555	47	242	5464.96	2265.76	6427.77	14158.48	1
14	0.05	0.10	0.07323	48	243	7207.09	2216.81	6317.52	15741.43	1
15	0.05	0.15	0.09046	49	245	8911.33	2179.30	6237.72	17328.35	1
16	0.10	0.01	0.04327	39	230	4272.26	4505.44	7379.54	16157.23	1
17	0.10	0.03	0.05093	39	230	5035.46	4494.43	7379.18	16909.06	1
18	0.10	0.05	0.05851	40	232	5754.18	4413.49	7255.49	17423.16	1
19	0.10	0.10	0.07709	40	232	7620.32	4386.83	7254.62	19261.77	1
20	0.10	0.15	0.09516	41	234	9398.32	4294.57	7136.47	20829.36	1
21	0.15	0.01	0.04573	33	219	4460.54	7206.35	8360.51	20027.40	1
22	0.15	0.03	0.05381	33	219	5255.17	7187.55	8360.08	20802.80	1
23	0.15	0.05	0.06180	34	221	5989.59	7029.24	8185.83	21204.66	1
24	0.15	0.10	0.08136	34	221	7926.67	6983.91	8184.78	23095.36	1
25	0.15	0.15	0.10036	35	223	9750.23	6808.89	8019.89	24579.01	1

Table A.6 Effect of the Inspection Errors on the Inspection Plan of Model 2
Egr = Erg = Ers = Esr = 0.01

No.	Egs	Esg	1-PG	A(n)	ATI	TCFA	TCFR	TCI	ETC	n
1	0.01	0.01	0.00043	54	276	45.62	360.20	5097.53	5503.35	1
2	0.01	0.03	0.00121	55	278	129.61	354.79	5061.07	5545.47	1
3	0.01	0.05	0.00202	55	280	215.94	355.73	5130.93	5702.60	1
4	0.01	0.10	0.00414	55	284	449.57	357.47	5284.09	6091.12	1
5	0.01	0.15	0.00643	55	287	705.79	359.20	5437.75	6502.74	1
6	0.03	0.01	0.00044	51	272	46.72	1025.26	5355.72	6427.70	1
7	0.03	0.03	0.00124	52	275	132.68	1012.78	5329.48	6474.94	1
8	0.03	0.05	0.00206	52	275	221.09	1009.59	5384.79	6615.47	1
9	0.03	0.10	0.00425	52	279	460.79	1011.99	5545.23	7018.01	1
10	0.03	0.15	0.00659	52	282	724.20	1014.34	5706.25	7444.80	1
11	0.05	0.01	0.00045	48	269	47.95	1752.10	5665.14	7465.19	1
12	0.05	0.03	0.00127	48	270	136.31	1746.00	5724.48	7606.79	1
13	0.05	0.05	0.00214	49	272	226.83	1722.64	5689.18	7638.66	1
14	0.05	0.10	0.00435	49	275	473.34	1725.84	5858.84	8058.02	1
15	0.05	0.15	0.00676	49	279	744.77	1728.94	6029.14	8502.86	1
16	0.10	0.01	0.00047	40	258	51.31	3914.13	6683.66	10649.10	1
17	0.10	0.03	0.00135	41	260	145.43	3847.71	6615.43	10608.57	1
18	0.10	0.05	0.00225	41	262	242.51	3833.17	6682.92	10758.61	1
19	0.10	0.10	0.00465	41	265	507.98	3840.12	6880.06	11228.16	1
20	0.10	0.15	0.00723	41	269	802.16	3846.79	7078.19	11727.13	1
21	0.15	0.01	0.00051	35	250	54.02	6401.66	7543.45	13999.13	1
22	0.15	0.03	0.00144	35	251	153.64	6376.19	7621.22	14151.05	1
23	0.15	0.05	0.00241	34	252	263.88	6537.84	7924.42	14726.14	1
24	0.15	0.10	0.00498	35	255	539.08	6364.01	7923.57	14826.66	1
25	0.15	0.15	0.00777	34	258	859.67	6522.46	8379.50	15761.63	1

Table A.7 Effect of the Inspection Errors on the Inspection Plan of Model 2
Egr = Erg = Ers = Esr = 0.03

No.	Egs	Esg	1-PG	A(n)	ATI	TCFA	TCFR	TCI	ETC	n
1	0.01	0.01	0.00068	54	277	71.73	452.51	5259.78	5784.02	1
2	0.01	0.03	0.00167	54	278	175.31	452.96	5310.40	5938.67	1
3	0.01	0.05	0.00268	55	279	280.90	446.97	5268.87	5996.74	1
4	0.01	0.10	0.00531	55	283	564.35	451.76	5416.65	6432.77	1
5	0.01	0.15	0.00809	55	286	871.23	456.55	5564.91	6892.70	1
6	0.03	0.01	0.00070	51	273	73.78	1121.58	5521.31	6716.67	1
7	0.03	0.03	0.00172	52	276	180.00	1108.69	5491.97	6780.66	1
8	0.03	0.05	0.00275	52	276	288.57	1106.63	5544.18	6939.38	1
9	0.03	0.10	0.00546	52	280	580.20	1112.19	5699.97	7392.36	1
10	0.03	0.15	0.00832	52	283	896.44	1117.71	5856.30	7870.46	1
11	0.05	0.01	0.00072	48	269	76.06	1853.25	5835.78	7765.10	1
12	0.05	0.03	0.00176	48	271	185.63	1848.35	5891.79	7925.78	1
13	0.05	0.05	0.00283	49	273	297.11	1824.41	5853.16	7974.69	1
14	0.05	0.10	0.00579	49	277	597.86	1830.90	6017.94	8446.69	1
15	0.05	0.15	0.00856	49	280	924.52	1837.31	6183.30	8945.13	1
16	0.10	0.01	0.00078	40	259	82.40	4031.49	6867.53	10981.42	1
17	0.10	0.03	0.00190	41	261	200.06	3963.86	6795.09	10959.01	1
18	0.10	0.05	0.00305	41	262	320.63	3950.65	6858.79	11130.06	1
19	0.10	0.10	0.00605	41	265	646.91	3961.26	7050.43	11658.59	1
20	0.10	0.15	0.00924	41	269	1003.22	3971.63	7242.96	12217.81	1
21	0.15	0.01	0.00085	34	250	88.34	6680.25	7956.81	14725.39	1
22	0.15	0.03	0.00206	34	251	219.96	6699.04	8040.63	14959.62	1
23	0.15	0.05	0.00330	34	252	352.40	6674.65	8115.23	15142.28	1
24	0.15	0.10	0.00655	34	255	696.41	6649.92	8329.40	15675.73	1
25	0.15	0.15	0.01001	34	259	1083.80	6667.55	8554.74	16306.09	1

Table A.8 Effect of the Inspection Errors on the Inspection Plan of Model 2
Egr = Erg = Ers = Esr = 0.05

No.	Egs	Esg	1-PG	A(n)	ATI	TCFA	TCFR	TCI	ETC	n
1	0.01	0.01	0.00104	54	279	107.23	543.42	5400.66	6051.31	1
2	0.01	0.03	0.00223	54	280	229.85	547.21	5467.36	6244.41	1
3	0.01	0.05	0.00344	55	281	354.30	540.63	5421.87	6316.79	1
4	0.01	0.10	0.00656	55	285	686.48	548.50	5565.08	6800.07	1
5	0.01	0.15	0.00984	55	289	1042.83	556.38	5708.73	7307.95	1
6	0.03	0.01	0.00107	51	275	110.61	1218.74	5684.56	7013.91	1
7	0.03	0.03	0.00230	51	276	236.85	1217.81	5734.34	7189.00	1
8	0.03	0.05	0.00354	52	278	364.82	1204.54	5701.14	7270.49	1
9	0.03	0.10	0.00676	52	282	707.20	1213.30	5852.15	7772.64	1
10	0.03	0.15	0.01015	52	285	1075.00	1222.04	6003.63	8300.67	1
11	0.05	0.01	0.00111	48	271	114.38	1955.29	6003.94	8073.61	1
12	0.05	0.03	0.00237	48	272	244.63	1951.60	6056.58	8252.81	1
13	0.05	0.05	0.00365	48	273	377.10	1947.95	6108.61	8433.66	1
14	0.05	0.10	0.00698	48	277	731.65	1958.13	6269.52	8959.30	1
15	0.05	0.15	0.01047	48	279	1113.32	1968.25	6431.03	9512.59	1
16	0.10	0.01	0.00121	40	260	124.86	4149.84	7048.51	11323.21	1
17	0.10	0.03	0.00257	40	261	266.26	4137.75	7109.49	11513.50	1
18	0.10	0.05	0.00396	41	264	408.81	4069.15	7031.66	11509.62	1
19	0.10	0.10	0.00756	41	267	794.43	4083.50	7217.61	12095.54	1
20	0.10	0.15	0.01135	41	271	1211.28	4097.63	7404.36	12713.26	1
21	0.15	0.01	0.00132	34	251	134.95	6778.97	8119.54	15033.46	1
22	0.15	0.03	0.00280	34	252	286.87	6792.34	8220.32	15299.53	1
23	0.15	0.05	0.00432	34	253	452.26	6812.64	8302.52	15567.42	1
24	0.15	0.10	0.00824	34	256	860.15	6792.11	8507.23	16159.49	1
25	0.15	0.15	0.01237	34	259	1315.37	6813.97	8726.02	16855.35	1

Table A.9 Effect of the Inspection Errors on the Inspection Plan of Model 2
Egr = Erg = Ers = Esr = 0.10

No.	Egs	Esg	1-PG	A(n)	ATI	TCFA	TCFR	TCI	ETC	n
1	0.01	0.01	0.00236	53	281	235.39	791.78	5885.13	6912.31	1
2	0.01	0.03	0.00404	54	284	403.36	786.47	5849.56	7039.39	1
3	0.01	0.05	0.00575	53	284	583.97	805.78	5999.96	7389.71	1
4	0.01	0.10	0.01012	54	288	1022.85	806.82	6021.63	7851.30	1
5	0.01	0.15	0.01463	54	291	1498.49	822.83	6154.74	8476.06	1
6	0.03	0.01	0.00244	51	278	243.51	1465.36	6082.23	7791.10	1
7	0.03	0.03	0.00418	51	280	417.05	1467.40	6123.82	8008.27	1
8	0.03	0.05	0.00595	50	281	604.17	1505.24	6312.44	8421.84	1
9	0.03	0.10	0.01047	51	284	1057.19	1492.99	6328.84	8879.03	1
10	0.03	0.15	0.01512	52	289	1546.76	1487.09	6359.80	9393.65	1
11	0.05	0.01	0.00253	47	273	253.10	2240.68	6510.93	9004.71	1
12	0.05	0.03	0.00433	48	275	432.25	2213.71	6457.21	9103.17	1
13	0.05	0.05	0.00616	48	277	613.59	2213.12	6500.68	9327.39	1
14	0.05	0.10	0.01083	48	279	1095.35	2231.92	6648.35	9975.63	1
15	0.05	0.15	0.01565	48	283	1606.07	2250.70	6796.47	10653.24	1
16	0.10	0.01	0.00278	40	263	278.32	4450.20	7487.97	12216.48	1
17	0.10	0.03	0.00475	40	265	474.62	4441.60	7538.91	12455.13	1
18	0.10	0.05	0.00674	40	265	672.86	4433.10	7589.22	12695.18	1
19	0.10	0.10	0.01184	41	271	1198.10	4393.94	7621.36	13213.41	1
20	0.10	0.15	0.01710	41	274	1759.03	4417.73	7792.85	13969.62	1
21	0.15	0.01	0.00307	34	254	303.59	7120.42	8583.25	16007.26	1
22	0.15	0.03	0.00524	34	255	516.03	7138.74	8676.79	16331.56	1
23	0.15	0.05	0.00743	33	255	752.70	7335.77	8999.38	17087.86	1
24	0.15	0.10	0.01302	34	259	1307.01	7153.17	8935.19	17395.37	1
25	0.15	0.15	0.01880	34	262	1923.20	7185.90	9136.65	18245.75	1

Table A.10 Effect of the Inspection Errors on the Inspection Plan of Model 2
Egr = Erg = Ers = Esr = 0.15

No.	Egs	Esg	1-PG	A(n)	ATI	TCFA	TCFR	TCI	ETC	n
1	0.01	0.01	0.00428	53	286	415.99	1037.08	6260.10	7713.18	1
2	0.01	0.03	0.00647	53	286	626.68	1044.48	6291.84	7963.00	1
3	0.01	0.05	0.00867	53	289	853.88	1057.97	6365.93	8277.78	1
4	0.01	0.10	0.01426	54	292	1397.56	1062.60	6367.86	8828.01	1
5	0.01	0.15	0.01997	54	295	1984.03	1086.86	6487.90	9558.79	1
6	0.03	0.01	0.00444	50	281	431.32	1738.45	6549.50	8719.28	1
7	0.03	0.03	0.00670	51	283	648.42	1722.56	6497.58	8868.55	1
8	0.03	0.05	0.00898	50	284	884.63	1762.13	6660.92	9307.68	1
9	0.03	0.10	0.01477	51	287	1446.27	1753.35	6656.28	9855.90	1
10	0.03	0.15	0.02068	51	291	2053.76	1779.14	6782.51	10615.41	1
11	0.05	0.01	0.00461	47	278	448.35	2510.09	6903.86	9862.31	1
12	0.05	0.03	0.00696	47	278	674.08	2512.70	6939.03	10125.82	1
13	0.05	0.05	0.00932	47	280	918.81	2537.09	7022.08	10477.98	1
14	0.05	0.10	0.01531	48	284	1500.34	2511.96	7009.25	11021.55	1
15	0.05	0.15	0.02143	48	287	2131.20	2539.69	7143.02	11813.92	1
16	0.10	0.01	0.00509	39	266	496.13	4830.34	8054.19	13380.67	1
17	0.10	0.03	0.00766	40	268	741.51	4752.12	7948.88	13442.51	1
18	0.10	0.05	0.01025	40	268	989.94	4747.17	7988.93	13726.04	1
19	0.10	0.10	0.01682	40	272	1652.07	4782.17	8145.65	14579.89	1
20	0.10	0.15	0.02352	40	275	2350.06	4817.06	8302.87	15469.98	1
21	0.15	0.01	0.00565	34	257	540.88	7469.25	9024.99	17035.12	1
22	0.15	0.03	0.00849	33	257	833.44	7680.32	9346.18	17859.95	1
23	0.15	0.05	0.01134	33	259	1111.07	7704.60	9434.22	18249.89	1
24	0.15	0.10	0.01858	34	262	1800.99	7522.37	9338.84	18662.20	1
25	0.15	0.15	0.02596	34	265	2565.57	7566.43	9521.63	19653.63	1

APPENDIX (B)

PROGRAM LISTING FOR MODEL 1

```
C                                   **********
C                                   APPENDIX B
C                                   **********
C     ************************************************************
C     ************************************************************
C                      PROGRAM LISTING FOR MODEL 1
C     ************************************************************
C     ************************************************************

      integer N,Ca,ATI(40),ATII,zz
      real C1(10),C2(10),CI1(10),CI2(10),P(12,12),seq(12)
      real pg(10,0:9), pr(10,0:9), ps(10,0:9),TCFR(10),TCI(10)
      real egr(10), egs(10), erg(10),FA(10,10),TCFA(10),CAA(10,10)
      real ers(10), esg(10), esr(10),s(12,12),sss(10)
      real b(40),c(40),d(40),e(40),TC(40,40),TC1(40,40),T(40,40)
      real Etc(10),PGD,CI(10),R(10,10),f(10,10),PGO(12,12),Etc1(10)
      real PSS(10,10),CFR(10),CFA(10),CII(10),PRR(12,12)
      real cost1(10),cost2(10),error1(10),error2(10),error3(10)
      real error5(10),error6(10),prob1(20),prob2(20),error4(10)
      real Ma(10,10),Aa(10),M(10),PP(12,12),FGR(12,12)
      real ppst1(12),ppnd1(12),ipnd1(12),ipst1(12),pand1(12)
      real iiand1(10),t1(260,12),iast1(12),past1(12),ss1(12)
      real ppst2(12),ppnd2(12),ipnd2(12),ipst2(12),pand2(12)
      real iiand2(10),t2(260,12),iast2(12),past2(12),ss2(12)

      N=5
      M(1)=100
      do 3 iseq=1,25
      open(unit=12,file="probabilities5.txt",status="old")
      open(unit=15,file="costs5.txt",status="old")
      open(unit=16,file="cinsp5.txt",status="old")

      read(12,*)(T(iseq,k),k=1,10)
      read(15,*)(TC(iseq,k),k=1,6)
      read(16,*)(TC1(iseq,k),k=1,10)
3     continue
      close(12)
      close(15)
      close(15)

      do 300 iseq=1,40
      Ca=TC(iseq,1)
      Cfgs=TC(iseq,2)
      Cfrs=TC(iseq,3)

      do 4 i=1,N
      egs(i)=TC(iseq,4)
      esg(i)=TC(iseq,5)
      egr(i)=TC(iseq,6)
      erg(i)=TC(iseq,6)
      esr(i)=TC(iseq,6)
      ers(i)=TC(iseq,6)
4     continue

      do 5 k=1,N
      pr(k,0)=T(iseq,k)
      ps(k,0)=T(iseq,k+5)
```

```
 5      continue

        do 6 k=1,N
        C1(k)=TC1(iseq,k)
        C2(k)=TC1(iseq,k+5)
 6      continue

        egs(0)=0
        do 8 i=1,N
        cost1(i)=C1(i)
        cost2(i)=C2(i)
        prob1(i)=pr(i,0)
        prob2(i)=ps(i,0)
        error1(i)=egs(i)
        error2(i)=esg(i)
        error3(i)=egr(i)
        error4(i)=erg(i)
        error5(i)=ers(i)
        error6(i)=esr(i)
 8      continue

c     *****************************************************************
c                         COST WITH NO INSPECTION
c     *****************************************************************

        j=0
        PGD=1
        do 10 i=1,N
        PGD=PGD*(1-pr(i,0)-ps(i,0))
        seq(i)=i
 10     continue
        a=PGD
        Etc(0)=Ca*(1-PGD)
        write(*,*)'Etc(0)',Etc(0)
        write(*,*)'______________________________________________'

c     *****************************************************************
c       Step 1            START OF INSPECTION
c     *****************************************************************

 17     j=j+1
        do 18 i=1,N
        C1(i)=cost1(i)
        C2(i)=cost2(i)
        pr(i,0)=prob1(i)
        ps(i,0)=prob2(i)
        egs(i)=error1(i)
        esg(i)=error2(i)
        egr(i)=error3(i)
        erg(i)=error4(i)
        ers(i)=error5(i)
        esr(i)=error6(i)
 18     continue

        do 19 k=1,N
        seq(k)=k
 19     continue
```

```
      do 20 k=1,N
      pg(k,0)=(1-pr(k,0)-ps(k,0))
      b(k)=pr(k,0)*(erg(k))**(j-1)
      c(k)=pg(k,0)*(1-egs(k))**(j-1)
      d(k)=ps(k,0)*(esg(k))**(j-1)
      e(k)=b(k)+c(k)+d(k)
      pr(k,j)=b(k)/(c(k)+b(k)+d(k))
      ps(k,j)=d(k)/(c(k)+b(k)+d(k))
      pg(k,j)=c(k)/(c(k)+b(k)+d(k))
20    continue


c     ***************************************************************
c      Step 2          SEQUENCING THE CHARACTERISTICS
c     ***************************************************************
c                                Cost of Inspection

      do 25 k=1,N
      CI(k)=C1(k)+pr(k,j)*C2(k)
25    continue


c                                Rejection Rate

      do 30 k=1,N
      R(k,j)=ps(k,j)*(1-esg(k)-esr(k))+pg(k,j)*egs(k)
     &+pr(k,j)*ers(k)
30    continue


c                                Ratio
      do 35 k=1,N
      f(k,j)=CI(k)/R(k,j)
c     write(*,*)f(k,j)
35    continue

c                                Swapping

      do 40 k=1,N
      do 40 l=1,N-1
          if(f(l,j).gt.f(l+1,j))then
              call swap(f(l,j),f(l+1,j))
              call swap(seq(l),seq(l+1))
                  do 38 jj=0,j
                  call swap(pg(l,jj),pg(l+1,jj))
                  call swap(pr(l,jj),pr(l+1,jj))
                  call swap(ps(l,jj),ps(l+1,jj))
38                continue
              call swap(C1(l),C1(l+1))
              call swap(C2(l),C2(l+1))
              call swap(egr(l),egr(l+1))
              call swap(egs(l),egs(l+1))
              call swap(erg(l),erg(l+1))
              call swap(ers(l),ers(l+1))
              call swap(esg(l),esg(l+1))
              call swap(esr(l),esr(l+1))
          endif
```

```
 40     continue

        write(*,*)(seq(k),k=1,N)

 C     *****************************************************************

 c      STEP-3          Calculating PG(i,j)
 C     *****************************************************************


        do 200 mk=1,N
        PGO(mk,j)=1
            do 50 k=1,N
            PGO(mk,j)=PGO(mk,j)*(1-pr(k,j)-ps(k,j))
 50         continue

 c                       Calculating PR(i,j)

            z=1
            kq=1
            do 170 jx=1,N
                do 60 i=1,jx
                iix=N
                iiy=N-jx
                iiz=jx
                call fac(iix)
                call fac(iiy)
                call fac(iiz)
                zz=iix/(iiy*iiz)
                jjj=0
                ppst1(i)=pr(i,j)
                ipst1(i)=i
                past1(i)=ppst1(i)
                iast1(i)=ipst1(i)
                ppst2(i)=ps(i,j)
                ipst2(i)=i
                past2(i)=ppst2(i)
                iast2(i)=ipst2(i)
 60             continue
                do 70 i=1,N-jx
                ppnd1(i)=pr(jx+i,j)
                ipnd1(i)=i+jx
                pand1(i)=ppnd1(i)
                iiand1(i)=ipnd1(i)
                ppnd2(i)=ps(jx+i,j)
                ipnd2(i)=i+jx
                pand2(i)=ppnd2(i)
                iiand2(i)=ipnd2(i)
 70             continue
                k=1
                t1(k,jx)=1
                t2(k,jx)=1
                do 80 i=1,jx
                t1(k,jx)=t1(k,jx)*past1(i)
                t2(k,jx)=t2(k,jx)*past2(i)
 80             continue
                k=k+1
```

```
1000      tt=N-jx-jjj
          s=jx
          do 90 iy=1,tt
          iast1(s)=iiand1(iy)
          past1(s)=pand1(iy)
          iast2(s)=iiand2(iy)
          past2(s)=pand2(iy)
          t1(k,jx)=1
          t2(k,jx)=1
               do 85 i=1,jx
               t1(k,jx)=t1(k,jx)*past1(i)
               t2(k,jx)=t2(k,jx)*past2(i)
85             continue
          k=k+1
90        continue

          do 140 i=jx,1,-1
          if((iast1(i).eq.N).and.(iast1(1).lt.(N-jx+1)))then
               do 130 kk=jx-z,1,-1
               if(iast1(kk).lt.(N+kk-jx))then
               temp=iast1(kk)
                    do 100 l=kk,jx
                         if(l.eq.kk)then
                              iast1(l)=temp+1
                              iast2(l)=temp+1
                         else
                              iast1(l)=iast1(l-1)+1
                              iast2(l)=iast2(l-1)+1
                         endif
100                 continue
               endif
                    do 110 l=1,jx
                    past1(l)=pr(iast1(l),j)
                    past2(l)=ps(iast2(l),j)
110                 continue
               t1(k,jx)=1
               t2(k,jx)=1
                    do 120 ii=1,jx
                    t1(k,jx)=t1(k,jx)*past1(ii)
                    t2(k,jx)=t1(k,j)*past2(ii)
120                 continue
               k=k+1
130            continue
          endif
140       continue

          do 150 l=1,(N-iast1(jx))
          iiand1(l)=iast1(jx)+1
          pand1(l)=pr(iiand1(l),j)
          iiand2(l)=iast2(jx)+1
          pand2(l)=ps(iiand2(l),j)
150       continue
          jjj=jjj+1
          if((k.lt.(zz+1)).and.(iast1(jx-1).ne.(N-1)))then
               goto 1000
          else
          ss1(jx)=0
          ss2(jx)=0
```

```
                do 160 l=1,zz
                ss1(jx)=ss1(jx)+t1(l,jx)
                ss2(jx)=ss2(jx)+t2(l,jx)
 160            continue
             endif
 170  continue

         PRR(mk,j)=0
         do 180 mm=1,N
         PRR(mk,j)=PRR(mk,j)+((-1)**(mm+1))*ss1(mm)
 180     continue


c                    Calculating PS(i,j)

         PSS(mk,j)=1-PRR(mk,j)-PGO(mk,j)

C     ***************************************************************
c            Accomodating for the components defective
c               in more than one characteristic
C     ***************************************************************

        sss(2)=0
        do 182 k=1,mk-1
        sss(2)=sss(2)+pr(k,j)*pr(mk,j)+ps(k,j)*ps(mk,j)
 182    continue
        if(mk.gt.2)then
        sss(3)=0
           do 184 i=1,mk-2
           do 184 k=i+1,mk-1
           sss(3)=sss(3)+pr(i,j)*pr(k,j)*pr(mk,j)+ps(i,j)*ps(k,j)*
     &     ps(mk,j)
 184       continue
        sss(4)=0
           do 186 i=1,mk-3
           do 186 k=1,mk-2
           do 186 l=1,mk-1
           sss(4)=sss(4)+pr(i,j)*pr(k,j)*pr(l,j)*pr(mk,j)+ps(i,j)*
     &     ps(k,j)*ps(l,j)*ps(mk,j)
 186       continue
           do 188 k=5,N
           sss(k)=sss(4)
 188       continue
        endif
        PP(mk,j)=0
           do 190 im=2,mk
           PP(mk,j)=PP(mk,j)+((-1)**im)*(sss(im))
 190       continue

        P(mk,j)=1-pr(mk,j)-ps(mk,j)+PP(mk,j)

c                    Calculating M(j),M(i,j)
        If(j.gt.1)then
           M(j)=Aa(j-1)
        endif
        if(mk.eq.1)then
           Ma(mk,j)=M(j)
```

```
      endif

c                         Components Accepted

      FA(mk,j)=Ma(mk,j)*(pr(mk,j)*erg(mk)+ps(mk,j)*esg(mk)+
     &      (P(mk,j)-PGO(mk,j))*(1-egr(mk)-egs(mk)))
      CAA(mk,j)=Ma(mk,j)*PGO(mk,j)*(1-egr(mk)-egs(mk))
      FGR(mk,j)=Ma(mk,j)*PGO(mk,j)*egr(mk)
      Ma(mk+1,j)=FA(mk,j)+CAA(mk,j)+FGR(mk,j)

c                         Updating the Probabilities

      b(mk)=pr(mk,0)*(erg(mk))**j
      c(mk)=pg(mk,0)*(1-egs(mk))**j
      d(mk)=ps(mk,0)*(esg(mk))**j
      e(mk)=b(mk)+c(mk)+d(mk)
      pr(mk,j)=b(mk)/(c(mk)+b(mk)+d(mk))
      ps(mk,j)=d(mk)/(c(mk)+b(mk)+d(mk))

200   continue

C     ****************************************************************
c     STEP-4            Calculating A(j)
C     ****************************************************************

      Aa(j)=FA(N,j)+CAA(N,j)+FGR(N,j)
      write(*,*)'A(j)',Aa(j)
      write(*,*)'PG(j)',PG(1,j)

c                         Calculating CFR(j)

      x=0
      z=0
      do 220 i=1,N
      y=1
          do 210 k=0,i-1
210       y=y*(1-egs(k))
      z=z+y*egs(i)
      x=x+Ma(i,j)*PRR(i,j)*ers(i)
220   continue
      CFR(j)=0
      do 230 i=1,N
      CFR(j)=CFR(j)+Ma(i,j)*(PRR(i,j)*ers(i)*Cfrs+PGO(i,j)*egs(i)*Cfgs)
230   continue

c                         Calculating CFA(j)
      CFA(j)=Ca*FA(N,j)

c                         Calculating CI at the stage

      CI1(j)=0
```

```
      CI2(j)=0
      yy=0
      zz=0

      do 240 i=1,N
      CI1(j)=CI1(j)+C1(i)*Ma(i,j)
      CI2(j)=CI2(j)+C2(i)*Ma(i,j)*(PGO(i,j)*egr(i)+PSS(i,j)*esr(i)
     &+PRR(i,j)*(1-erg(i)-ers(i)))
      yy=yy+Ma(i,j)
      zz=zz+Ma(i,j)*(PGO(i,j)*egr(i)+PSS(i,j)*esr(i)
     &+PRR(i,j)*(1-erg(i)-ers(i)))

240   continue

c                        Total Cost of Inspection

      CII(j)=CI1(j)+CI2(j)
      ATI(j)=yy+zz

c     STEP-5             Components of Total Cost

      TA=Aa(j)
c                        TCFR
      x=0
      do 250 jk=1,j
250   x=x+CFR(jk)
      TCFR(j)=x/TA
      write(*,*)'TCFR(j)',TCFR(j)

c                        TCFA
      TCFA(j)=CFA(j)/TA
      write(*,*)'TCFA(j)',TCFA(j)

c                        TCI
      x=0
      do 260 jj=1,j
260   x=x+CII(jj)
      TCI(j)=x/TA
      write(*,*)'TCI(j)',TCI(j)

c     STEP-6             Expected Total Cost

      Etc(j)=TCFR(j)+TCFA(j)+TCI(j)
      Etc1(j)=(TCFR(j)+TCFA(j)+TCI(j))*TA
      write(*,*)'Etc(j)',Etc(j)
      write(*,*)'_____________________________________________'

c     STEP-7             Comparison of Cost
```

```
      if(j.eq.1) then
          goto 17
      endif
      if(Etc(j).lt.Etc(j-1)) then
          goto 17
      else
          nnx=j-1
          zzz=1-PGO(1,j)
      endif
      ATII=0
      do 270 k=1,nnx
      ATII=ATII + ATI(k)
270   continue
      write(*,280)nnx

300   continue
      end

      subroutine swap(a,b)
      t=a
      a=b
      b=t
      end

      subroutine fac(iik)
      ix=1
      do 290 i=iik,1,-1
      ix=ix*i
290   continue
      iik=ix
      end
```

APPENDIX (C)

PROGRAM LISTING FOR MODEL 2

```
C                          **********
C                          APPENDIX C
C                          **********
C    ****************************************************************
C    ****************************************************************
C                  PROGRAM LISTING FOR MODEL 2
C    ****************************************************************
C    ****************************************************************

      integer N,Ca,zz(5),ATI(10),ATII,o,pp
      real C1(10),C2(10),CI(10),P(5,5,5),PG(5,5),PRR(5,5),PS(5,5)
      real egr(10), egs(10), erg(10),pr(5,5,3),R(5,5),FA(5,5),FGR(5,5)
      real ers(10), esg(10), esr(10),Etc(5),seq(5),f(5,5),CAA(5,5)
      real TCI(10),TCFA(10),TCFR(10),cc(5),PPP(5,5),fff(5),hhh(5)
      real cost1(5),cost2(5),error1(5),error2(5),error3(5),error4(5)
      real error5(5),error6(5),M(5),Mn(5,5),A(5)
      real CFA(5),CFR(5),CI1(5,5),CI2(5,5),CFRR(5,5),CII(10)
      open(unit=5,file="d21.txt",status="old")
      read(5,*)N
      read(5,*)Ca

      read(5,*)M(2)
      read(5,*)(C1(i),i=1,N)
      read(5,*)(C2(i),i=1,N)
      do 10 i=1,3
      do 10 j=1,3
      read(5,*)(P(i,j,k),k=1,3)
10    continue

      read(5,*)(egs(i),i=1,N)
      read(5,*)(esg(i),i=1,N)
      read(5,*)(egr(i),i=1,N)
      read(5,*)(erg(i),i=1,N)
      read(5,*)(ers(i),i=1,N)
      read(5,*)(esr(i),i=1,N)
      read(5,*)Cfgs
      read(5,*)Crs
      close(5)

      do 8 i=1,N
      cost1(i)=C1(i)
      cost2(i)=C2(i).
      error1(i)=egs(i)
      error2(i)=esg(i)
      error3(i)=egr(i)
      error4(i)=erg(i)
      error5(i)=ers(i)
      error6(i)=esr(i)
8     continue

C    ****************************************************************

c                              Cost of no inspection
C    ****************************************************************


      j=1
      Etc(1)=Ca*(1-P(3,3,3))
```

```
      write(*,*)'Etc(0)',Etc(1)

      write(*,*)'________________________________________________________'

C     ****************************************************************
c                                Start of inspection j=1
C     ****************************************************************

 11   j=j+1
      do 12 i=1,N
      C1(i)=cost1(i)
      C2(i)=cost2(i)
      egs(i)=error1(i)
      esg(i)=error2(i)
      egr(i)=error3(i)
      erg(i)=error4(i)
      ers(i)=error5(i)
      esr(i)=error6(i)
 12   continue

      ATI(j)=0
      do 13 kk=1,N
      seq(kk)=kk
 13   continue
      pp=1
 15   o=pp


      do 20 i=1,N

      do 18 l=1,3
      pr(i,o,l)=0

      do 17 k=1,3
      do 17 nn=1,3
          if(i.eq.1)then
          ix=l
          iy=k
          iz=nn
          pr(i,o,l)=pr(i,o,l)+P(ix,iy,iz)
          elseif(i.eq.2)then
          iy=l
          ix=k
          iz=nn
          pr(i,o,l)=pr(i,o,l)+P(ix,iy,iz)
          else
          iz=l
          iy=k
          ix=nn
          pr(i,o,l)=pr(i,o,l)+P(ix,iy,iz)
          endif
 17   continue
 18   continue
 20   continue

      if(o.ge.2)then
          do 75 k=1,N
```

```
        hhh(k)=fff(k)
 75     continue
        do 21 k=1,N-1
        do 21 l=N,k+1,-1
            if(hhh(l).lt.hhh(l-1))then
            call swap(pr(l,o,1),pr(l-1,o,1))
            call swap(hhh(l),hhh(l-1))
            call swap(pr(l,o,2),pr(l-1,o,2))
            call swap(pr(l,o,3),pr(l-1,o,3))
            endif
 21     continue
      endif

C     ******************************************************************
c                       Sequencing the Characteristics
C     ******************************************************************
c                                       Cost of Inspection

      do 25 k=o,N
      CI(k)=C1(k)+pr(k,o,2)*C2(k)

c     write(*,*)CI(k)
25    continue


c                                       Rejection Rate

      do 30 k=o,N
      R(k,j)=pr(k,o,1)*(1-esg(k)-esr(k))+pr(k,o,2)*ers(k)+pr(k,o,3)*
     &egs(k)
30    continue


c                                       Ratio
      do 35 k=o,N
      f(k,j)=CI(k)/R(k,j)
35    continue
      if(o.eq.1)then
      fff(1)=f(1,j)
      fff(2)=f(2,j)
      fff(3)=f(3,j)
      endif


c                                       Swapping

      do 39 k=o,N-1
      do 39 l=N,k+1,-1

          if(f(l,j).lt.f(l-1,j))then
          call swap(f(l,j),f(l-1,j))
          call swap(seq(l),seq(l-1))

          call swap(pr(l,o,1),pr(l-1,o,1))
          call swap(pr(l,o,2),pr(l-1,o,2))
38        call swap(pr(l,o,3),pr(l-1,o,3))

          call swap(C1(l),C1(l-1))
```

```
           call swap(C2(l),C2(l-1))
           call swap(egr(l),egr(l-1))
           call swap(egs(l),egs(l-1))
           call swap(erg(l),erg(l-1))
           call swap(ers(l),ers(l-1))
           call swap(esg(l),esg(l-1))
           call swap(esr(l),esr(l-1))
           endif

 39    continue
       do 40 k=1,N
 40    continue
       ii=seq(o)
       zz(o)=ii
       do 41 k=1,3
 41    continue


C     ****************************************************************
c                         Analysing the first stage
C     ****************************************************************

       if(o.eq.1)then
       Mn(o,j)=M(j)
c      write(*,*)'PG(j)',PG(1,j)
       else
       Mn(o,j)=FA(o-1,j)+CAA(o-1,j)+FGR(o-1,j)
       endif
       PG(o,j)=P(3,3,3)
       sum1=0
       sum2=0
       sum3=0
       do 42 kk=1,3
       do 42 ll=1,3
       sum1=sum1+P(2,kk,ll)
       sum2=sum2+P(kk,2,ll)
       sum3=sum3+P(kk,ll,2)
 42    continue
       sum4=0
       sum5=0
       sum6=0
       do 43 kk=1,3
       sum4=sum4+P(2,2,kk)
       sum5=sum5+P(2,kk,2)
       sum6=sum6+P(kk,2,2)
 43    continue

       PRR(o,j)=sum1+sum2+sum3-sum4-sum5-sum6+P(2,2,2)
       PS(o,j)=1-PG(o,j)-PRR(o,j)

       if(o.eq.1)then
       PPP(o,j)=1-pr(1,o,2)-pr(1,o,1)
       elseif(o.eq.2)then
       PPP(o,j)=1-pr(2,o,2)-pr(2,o,1)+pr(1,o,2)*pr(2,o,2)
     &+pr(1,o,1)*pr(2,o,1)
       else
       PPP(o,j)=1-pr(3,o,2)-pr(3,o,1)+pr(1,o,2)*pr(3,o,2)
     &+pr(2,o,2)*pr(3,o,2)-pr(1,o,2)*pr(2,o,2)*pr(3,o,2)+pr(1,o,1)
```

```
     &*pr(3,o,1)+pr(2,o,1)*pr(3,o,1)-pr(1,o,1)*pr(2,o,1)*pr(3,o,1)
      endif
      FA(o,j)=Mn(o,j)*(pr(o,o,2)*erg(o)+pr(o,o,1)*esg(o)+
     &(PPP(o,j)-PG(o,j))*(1-egr(o)-egs(o)))
      FGR(o,j)=Mn(o,j)*PG(o,j)*egr(o)
      CAA(o,j)=Mn(o,j)*PG(o,j)*(1-egs(o)-egr(o))
      CFRR(o,j)=Mn(o,j)*(PRR(o,j)*ers(o)*Crs+PG(o,j)*egs(o)*Cfgs)
      CI1(o,j)=C1(o)*Mn(o,j)
      CI2(o,j)=C2(o)*Mn(o,j)*(PG(o,j)*egr(o)+PS(o,j)*esr(o)
     &+PRR(o,j)*(1-erg(o)-ers(o)))
      ATI(j)=ATI(j)+ Mn(o,j)*(1+(PG(o,j)*egr(o)+PS(o,j)*esr(o)
     &+PRR(o,j)*(1-erg(o)-ers(o))))


C     *****************************************************************

c                       Updating the marginal probabilities
C     *****************************************************************

      pp=o+1
      pr(o,pp,1)=(pr(o,o,1)*esg(o))/(pr(o,o,3)*(1-egs(o))
     &+pr(o,o,1)*esg(o)+pr(o,o,2)*erg(o))
      pr(o,pp,2)=(pr(o,o,2)*erg(o))/(pr(o,o,3)*(1-egs(o))
     &+pr(o,o,1)*esg(o)+pr(o,o,2)*erg(o))
      pr(o,pp,3)=1-pr(o,pp,1)-pr(o,pp,2)


C     *****************************************************************

c                       Updating the joint probabilities
C     *****************************************************************

      if(ii.eq.1)then
          do 45 l=1,3
          do 45 k=1,3
          do 45 nn=1,3
          if(l.eq.1)then
               P(l,k,nn)=P(l,k,nn)*(pr(o,pp,1)/pr(o,o,1))
               elseif(l.eq.2)then
               P(l,k,nn)=P(l,k,nn)*(pr(o,pp,2)/pr(o,o,2))
               else
               P(l,k,nn)=P(l,k,nn)*(pr(o,pp,3)/pr(o,o,3))
               endif
45        continue
      elseif(ii.eq.2)then
          do 47 k=1,3
          do 47 nn=1,3
          do 46 l=1,3
               if(k.eq.1)then
               P(l,k,nn)=P(l,k,nn)*(pr(o,pp,1)/pr(o,o,1))
c              write(*,*)(P(l,k,nn))
               elseif(k.eq.2)then
               P(l,k,nn)=P(l,k,nn)*(pr(o,pp,2)/pr(o,o,2))
c              write(*,*)(P(l,k,nn))
               else
               P(l,k,nn)=P(l,k,nn)*(pr(o,pp,3)/pr(o,o,3))
c              write(*,*)(P(l,k,nn))
               endif
```

```
 46        continue
 47        continue
      else
           do 48 nn=1,3
           do 48 l=1,3
           do 48 k=1,3
                if(nn.eq.1)then
                P(l,k,nn)=P(l,k,nn)*(pr(o,pp,1)/pr(o,o,1))
                elseif(nn.eq.2)then
                P(l,k,nn)=P(l,k,nn)*(pr(o,pp,2)/pr(o,o,2))
                else
                P(l,k,nn)=P(l,k,nn)*(pr(o,pp,3)/pr(o,o,3))
                endif
 48        continue
           do 49 l=1,3
           do 49 k=1,3
 49        continue
      endif
      if(o.eq.N)then
           A(j)=FA(N,j)+CAA(N,j)+FGR(N,j)
           M(j+1)=A(j)
           aaa=0
           bbb=0
           ccc=0
           do 50 i=1,N
           aaa=aaa+CFRR(i,j)
           bbb=bbb+CI1(i,j)
           ccc=ccc+CI2(i,j)
c          write(*,*)ccc
 50        continue
           CFR(j)=aaa
           CII(j)=bbb+ccc
           CFA(j)=Ca*FA(N,j)


C     ****************************************************************
c                                Total Cost
C     ****************************************************************

           x=0
           y=0
           do 52 jj=1,j
           x=x+CFR(jj)
           y=y+CII(jj)
 52        continue
           TCFR(j)=x/A(j)
           TCI(j)=y/A(j)
           TCFA(j)=CFA(j)/A(j)
           TA=A(j)

           Etc(j)=TCFA(j)+TCFR(j)+TCI(j)
           cc(j)=PG(1,j)
           write(*,*)'PG(j)',cc(j)
           write(*,*)'A(j)',A(j)
           write(*,*)'Etc(j)',Etc(j)
           write(*,*)'_______________________________________________'

C     ****************************************************************
```

```
c                              Comparison of Cost
c     *****************************************************************

           if(j.eq.1)then
               goto 11
           endif
           if(Etc(j).lt.Etc(j-1))then
               goto 11
           else
               goto 55
           endif
       else
           goto 15
       endif

55     n=j-1
       zzz=1-cc(j)
       ATII=0
       do 58 k=1,n
       ATII=ATII+ATI(k)
58     continue
       nn=j-2
       write(*,*)'Optimal no. of Inspection',nn

       end

       subroutine swap(a,b)
       t=a
       a=b
       b=t
       end
```

APPENDIX (D)

PROGRAM LISTING FOR RANDOM PROBLEM GENERATION

```
c                        **********
c                        APPENDIX D
c                        **********
c     ****************************************************************
c     ****************************************************************
c          PROGRAM LISTING FOR RANDOM PROBLEM GENERATION
c     ****************************************************************
c     ****************************************************************

      real u1,u2,v1,v2,w,y,egs(100),esg(100),e(100)
      real pr(5),ps(5)
      integer Ci1(100),Ci2(100),Ca(100),Cfgs(100),Cfrs(100)
      integer seed
c     ***************************************************************

c                    INITIALIZING THE PROBLEM

c     ***************************************************************
      seed=200*secnds(0)
      do 120 j=1,25

c     ***************************************************************
c
c            Ca(j)=COST OF INSPECTION AT THE INSPECTION STATION IS
c                  ASSUMED TO HAVE A UNIFORM DISTRIBUTION BETWEEN a
c                  AND b.
c     ***************************************************************
      a=100000.0
      b=1000000.0
      u1=ran(seed*100*j)
      Ca(j)=a+(b-a)*u1

c     ***************************************************************
c
c            Ci1=COST OF INSPECTION AT THE INSPECTION STATION IS
c                ASSUMED TO HAVE A UNIFORM DISTRIBUTION BETWEEN a
c                AND b.
c     ***************************************************************
      do 30 i=1,5
      a=10.0
      b=100.0
c     u1=uniform(a,b)
      u1=ran(seed*200*i*j)
      Ci1(i)=a+(b-a)*u1
30    continue

c     ***************************************************************
c
c            Ci2=COST OF INSPECTION AT THE REWORK STATION IS
c                ASSUMED TO HAVE A UNIFORM DISTRIBUTION BETWEEN a
c                AND b.
c     ***************************************************************
      do 40 i=1,5
      a=10.0
      b=100.0
      u1=ran(seed*300*i*j)
      Ci2(i)=a+(b-a)*u1
40    continue
```

```
c     ****************************************************************
c
c              Cfgs=COST OF FALSELY CLASSIFYING A GOOD COMPONENT TO BE
c                   SCRAP IS ASSUMED TO HAVE A UNIFORM DISTRIBUTION
c                   BETWEEN a AND b.
c     ****************************************************************
       a=500.0
       b=1000.0
       u1=ran(seed*400*j)
       Cfgs(j)=a+(b-a)*u1

c     ****************************************************************
c
c              Cfrs=COST OF FALSELY CLASSIFYING A REWORK COMPONENT TO
c                   BE SCRAP IS ASSUMED TO HAVE A UNIFORM DISTRIBUTION
c                   BETWEEN a AND b.
c     ****************************************************************
       a=500.0
       b=1000.0
       u1=ran(seed*200*j)
       Cfrs(j)=a+(b-a)*u1

c     ****************************************************************
c
c              egs(j)=PROBABILITY OF CLASSIFYING A GOOD COMPONENT TO
c                   BE SCRAP IS ASSUMED TO HAVE A NORMAL DISTRIBUTION
c                   WITH MEAN 0.1 AND VARIANCE (0.03)**2
c     ****************************************************************
45     u1=ran(seed)
       u2=ran(seed)
       v1=2*u1-1
       v2=2*u2-1
       w=v1**2+v2**2
       if(w.gt.1.0)then
       goto 45
       else
       y=((-2*log(w))/w)**0.5
       endif
       x1=v1*y
       x2=v2*y
       if(mod(i,2).eq.0.0)then
       egs(j)=0.1+0.03*x2
       else
       egs(j)=0.10+0.03*x1
       endif

c     ****************************************************************
c
c              esg(j)=PROBABILITY OF CLASSIFYING A SCRAP COMPONENT TO
c                   BE GOOD IS ASSUMED TO HAVE A NORMAL DISTRIBUTION
c                   WITH MEAN 0.1 AND VARIANCE (0.03)**2
c     ****************************************************************
55     u1=ran(seed)
       u2=ran(seed)
       v1=2*u1-1
       v2=2*u2-1
       w=v1**2+v2**2
```

```
      if(w.gt.1.0)then
      goto 55
      else
      y=((-2*log(w))/w)**0.5
      endif
      x1=v1*y
      x2=v2*y
      if(mod(i,2).eq.0.0)then
      esg(j)=0.10+0.03*x2
      else
      esg(j)=0.10+0.03*x1
      endif

c    ***************************************************************
c
c                e(j)=PROBABILITY OF OTHER MISCLASSIFICATIONS
c                     IS ASSUMED TO HAVE A NORMAL DISTRIBUTION
c                     WITH MEAN 0.1 AND VARIANCE (0.03)**2
c    ***************************************************************
65    u1=ran(seed)
      u2=ran(seed)
      v1=2*u1-1
      v2=2*u2-1
      w=v1**2+v2**2
      if(w.gt.1.0)then
      goto 65
      else
      y=((-2*log(w))/w)**0.5
      endif
      x1=v1*y
      x2=v2*y
      if(mod(i,2).eq.0.0)then
      e(j)=0.10+0.03*x2
      else
      e(j)=0.10+0.03*x1
      endif

c    ***************************************************************
c
c                pr(i)=PROBABILITY OF A COMPONENT TO BE REWORKED
c                     IS ASSUMED TO HAVE A NORMAL DISTRIBUTION
c                     WITH MEAN 0.12 AND VARIANCE (0.03)**2
c    ***************************************************************
      do 80 i=1,5
75    u1=ran(seed)
      u2=ran(seed)
      v1=2*u1-1
      v2=2*u2-1
      w=v1**2+v2**2
      if(w.gt.1.0)then
      goto 75
      else
      y=((-2*log(w))/w)**0.5
      endif
      x1=v1*y
      x2=v2*y
      if(mod(i,2).eq.0.0)then
      pr(i)=0.05+0.03*x2
```

```
      else
      pr(i)=0.05+0.03*x1
      endif
 80   continue
 c    ***************************************************************
 c
 c                  ps(i)=PROBABILITY OF ith CHARACTERISTIC TO BE REWORKED
 c                        IS ASSUMED TO HAVE A NORMAL DISTRIBUTIONWITH MEAN
 c                        0.12 AND VARIANCE (0.03)**2
 c    ***************************************************************
      do 90 i=1,5
 85   u1=ran(seed)
      u2=ran(seed)
      v1=2*u1-1
      v2=2*u2-1
      w=v1**2+v2**2
      if(w.gt.1.0)then
      goto 75
      else
      y=((-2*log(w))/w)**0.5
      endif
      x1=v1*y
      x2=v2*y
      if(mod(i,2).eq.0.0)then
      ps(i)=0.05+0.03*x2
      else
      ps(i)=0.05+0.03*x1
      endif
 90   continue
 c    ***************************************************************

 c                    PRINTING THE RESULTS

 c    ***************************************************************
      open(unit=25,file="costs5.txt",status="unknown",access="append")
      open(unit=26,file="probabilities5.txt",status="unknown"
     &,access="append")
      open(unit=27,file="cinsp5.txt",status="unknown",access="append")
      write(25,10)Ca(j),Cfgs(j),Cfrs(j),egs(j),esg(j),e(j)
      write(26,20)(pr(i),i=1,5),(ps(i),i=1,5)
      write(27,35)(Ci1(i),i=1,5),(Ci2(i),i=1,5)
 10   format(i6,2x,i4,3x,i4,2x,3(f4.2,2x))
 20   format(10(f4.2,2x),2x,10(f4.2,2x))
 35   format(10(i4,2x),2x,10(i6,2x))

      close(25)
      close(26)
      close(27)
 120  continue
      end
```

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Vita

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